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Emergence of non-ergodic, conservative dynamics

The Birkhoff ergodic theorem states that given an ergodic probability measure μ, for μ-almost every point x, the Birkhoff average:

$$S_k(x) := \frac{1}{k}\sum_{i=1}^{k} \delta_{f^i(x)}$$

converges to μ. For many differentiables maps f, the Birkhoff average pushes forward the Lebesgue measure to a finite number of ergodic probability measures. On the other hand, Newhouse showed in the 70's that a locally generic diffeommorphism may display infinitely many attractors with very different ergodic behavior. I showed that Newhouse phenomena is furthermore locally typical in the sense of Kolmogorov. To describe the complexity of such dynamics, I introduced the following:

Definition: The **Emergence** $\mathcal{E}(\epsilon)$ at scale $\epsilon > 0$ of a system is the minimal number N of probability measures $(\mu_i)_{1\le i\le N}$ necessarily so that the Birkhoff average satisfies:

$$\limsup_{k\to\infty} \int_M d_{W_1}(S_k(x), \{\mu_i : 1 \le i \le N\}) dLeb < \epsilon \ ,$$

where d_{W_1} is the 1-Wasserstein metric on the space of probability measures of M.

Conjecture (B. 2017): In many categories of dynamical systems, a typical dynamics displays a super polynomial emergence: for every $k > 0$, $\lim_{\epsilon\to 0} \mathcal{E}(\epsilon) \cdot \epsilon^k \to \infty$.

We will present recent developments on this program, and its analog with the theory of systems of positive entropy.

In a work in progress with Jairo Bochi, we showed an analog of the variational principle of the entropy for the concept of emergence. With Turaev, we showed furthermore that in the open set of symplectomorphisms with a totally elliptic periodic point, a typical diffeomorphism in the sense of Kolmogorov (i.e. Leb. a.e. map in a generic family) displays a maximal, super exponential emergence.