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Operator error estimates for homogenization of periodic hyperbolic systems

In $L_2(\mathbb{R}^d;\mathbb{C}^n)$, we consider a matrix elliptic second order differential operator $A_\varepsilon = A_\varepsilon^* \geqslant 0$ given in a factorized form. The coefficients of the operator A_ε are periodic and depend on $\mathbf{x}/\varepsilon$, $0<\varepsilon\leqslant 1$. So, they oscillate rapidly as $\varepsilon \to 0$.

First result is the estimate

$$\|A_\varepsilon^{-1/2}\sin(tA_\varepsilon^{1/2}) - (A^0)^{-1/2}\sin(t(A^0)^{1/2})\|_{H^1(\mathbb{R}^d)\to L_2(\mathbb{R}^d)} \leqslant C_1\varepsilon(1+|t|).$$

Here A^0 is the effective operator with constant coefficients. The main result is approximation with the corrector $K(\varepsilon;t)$:

$$\|A_\varepsilon^{-1/2}\sin(tA_\varepsilon^{1/2}) - (A^0)^{-1/2}\sin(t(A^0)^{1/2}) - \varepsilon K(\varepsilon;t)\|_{H^2(\mathbb{R}^d)\to H^1(\mathbb{R}^d)} \leqslant C_2\varepsilon(1+|t|).$$

The constants C_1 and C_2 are controlled explicitly. The results of this type are called operator error estimates.

Results are applied to homogenization of periodic hyperbolic systems

$$\begin{cases} \partial_t^2\mathbf{u}_\varepsilon(\mathbf{x},t) = -A_\varepsilon \mathbf{u}_\varepsilon(\mathbf{x},t), & \mathbf{x}\in\mathbb{R}^d,\ t\in\mathbb{R}; \\ \mathbf{u}_\varepsilon(\mathbf{x},0)=0, \quad (\partial_t\mathbf{u}_\varepsilon)(\mathbf{x},0) = \boldsymbol{\psi}(\mathbf{x}), & \mathbf{x}\in\mathbb{R}^d, \end{cases}$$

where $\boldsymbol{\psi}\in H^1(\mathbb{R}^d;\mathbb{C}^n)$ or $\boldsymbol{\psi}\in H^2(\mathbb{R}^d;\mathbb{C}^n)$. Then $\mathbf{u}_\varepsilon(\cdot,t) = A_\varepsilon^{-1/2}\sin(tA_\varepsilon^{1/2})\boldsymbol{\psi}$ and

$$\begin{aligned} \|\mathbf{u}_\varepsilon(\cdot,t) - \mathbf{u}_0(\cdot,t)\|_{L_2(\mathbb{R}^d)} &\leqslant C_1\varepsilon(1+|t|)\|\boldsymbol{\psi}\|_{H^1(\mathbb{R}^d)}, \\ \|\mathbf{u}_\varepsilon(\cdot,t) - \mathbf{v}_\varepsilon(\cdot,t)\|_{H^1(\mathbb{R}^d)} &\leqslant C_2\varepsilon(1+|t|)\|\boldsymbol{\psi}\|_{H^2(\mathbb{R}^d)}. \end{aligned}$$

Here $\mathbf{u}_0$ is the solution of the effective problem and $\mathbf{v}_\varepsilon = \mathbf{u}_0 + \varepsilon K(\varepsilon;t)\boldsymbol{\psi}$ is the first order approximation.

We use the spectral approach to homogenization problems developed by M. Sh. Birman and T. A. Suslina. The method is based on the scaling transformation, the Floquet-Bloch theory and analytic perturbation theory. It turns out that homogenization is a spectral threshold effect at the bottom of the spectrum.

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