ABDELMALEK ABDESSELAM, University of Virginia

A proof of weak universality for a non-Gaussian scaling limit with anomalous scaling

Wilson's theory of critical phenomena provides a heuristic yet compelling explanation for the mechanism behind non-Gaussian scaling limits with anomalous scaling dimensions and the associated universality property. Such a scaling limit corresponds to a nontrivial renormalization group fixed point and the basin of attraction featuring in the universality property is the stable manifold of that fixed point. I will review results of joint work with A. Chandra and G. Guadagni where we turned Wilson's ideas into rigorous mathematical theorems. The model we studied is a hierarchical 3D ferromagnet with continuous spins, slightly below the upper critical dimension. The scaling limit is for the joint law of the spin field and its square. While the spin field does not exhibit anomalous scaling, we proved that the square/energy field does, which confirms a prediction made by Wilson in 1972. The (joint) scaling limit is shown to be the same for a large class of (irrelevant) interactions ϕ^6, ϕ^8, etc. However, this universality result which we proved is in the "weak sense", namely, these interactions are assumed to be small. Our result is based on a new rigorous renormalization group method which allows couplings to be space-dependent.