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Dynamics of the box-ball system with random initial conditions via Pitman's transformation

The box-ball system (BBS), introduced by Takahashi and Satsuma in 1990, is a cellular automaton that exhibits solitonic behaviour. In this talk, we study the BBS when started from a random two-sided infinite particle configuration. For such a model, Ferrari et al. recently showed the invariance in distribution of Bernoulli product measures with density strictly less than $\frac{1}{2}$, and gave a soliton decomposition for invariant measures more generally. We study the BBS dynamics using the transformation of a nearest neighbour path encoding of the particle configuration given by 'reflection in the past maximum', which was famously shown by Pitman to connect Brownian motion and a three-dimensional Bessel process. We use this to characterise the set of configurations for which the dynamics are well-defined and reversible for all times. We give simple sufficient conditions for random initial conditions to be invariant in distribution under the BBS dynamics, which we check in several natural examples, and also investigate the ergodicity of the relevant transformation. Furthermore, we analyse various probabilistic properties of the BBS that are commonly studied for interacting particle systems, such as the asymptotic behavior of the integrated current of particles and of a tagged particle. The talk is based on a joint work with David Croydon, Tsuyoshi Kato and Satoshi Tsujimoto.