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On the nodal distribution of quantum graphs

The nodal surplus is the suitably normalized number of zeros of a Laplacian eigenfunction. Berkolaikos remarkable nodal-magnetic connection implies that the nodal surplus of a metric graph eigenfunction is equal to the stability index of its eigenvalue under magnetic perturbations. It has been suggested that the distribution of the nodal surplus contains information about the geometry of the underlying domain. We study the nodal surplus distribution for metric graphs. The existence of the distribution is established, along with its symmetry. One consequence of the symmetry is that a topological quantity - the Euler characteristic of the graph can be recovered as twice the average nodal surplus of its eigenfunctions. Furthermore, for graphs with disjoint cycles it is proven that the distribution has a universal form - it is binomial over the allowed range of values of the surplus.