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Explicit Construction of the Voronoi and Delaunay Cells of W(An) and W(Dn) Lattices and Their Facets

Voronoi and Delaunay (Delone) cells of the root and weight lattices of the Coxeter-Weyl groups W(an) and W(dn) are constructed. The face centered cubic (fcc) and body centered cubic (bcc) lattices are obtained in this context. Basic definitions are introduced such as parallelotope, fundamental simplex, contact polytope, root polytope, Voronoi cell, Delone cell, n-simplex, n-octahedron (cross polytope), n-cube and n-hemicube and their volumes are calculated. Voronoi cell of the root lattice is constructed as the dual of the root polytope which turns out to be the union of Delone cells. It is shown that the Delone cells centered at the origin of the root lattice An are the polytopes of the fundamental weights w1, w2,..., wn and the Delone cells of the root lattice Dn are the polytopes obtained from the weights w1, wn-1, wn. A simple mechanism explains the tessellation of the root lattice by Delone cells. We prove that the (n-1) facet of the Voronoi cell of the root lattice An is (n-1) -dimensional rhombohedron and similarly the (n-1) - facet of the Voronoi cell of the root lattice Dn is a dipyramid with a base of (n-2) cube. Volume of the Voronoi cell is calculated via its (n-1) -facet which in turn can be obtained from the fundamental simplex. Tessellations of the root lattice with the Voronoi and Delone cells are explained by giving examples from lower dimensions. Similar considerations are also worked out for the weight lattices An* and Dn*.