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An Analytic Approach to Quantum Shannon Theory

We characterize the conditions under which the quantum channel capacities are analytic. If the optimization over input states is restricted to positive operators, the quantum and classical channel capacities are analytic on the set of channels. This proof arises naturally from Danskin's Theorem which characterizes the Gatiéaux differential of an optimization function and a number of standard theorems from complex function theory. We calculate the general power series expansion of the n-shot classical and quantum capacity using standard techniques from functional calculus. This can be used to directly compute the n-shot quantum capacity without any optimization given that the center of the expansion is a degradable channel. As a direct application of computing these derivatives, we develop a necessary and sufficient criterion for n-shot superactivation. It is also shown that the power series expansion can be inverted using Lagrange's Inversion Theorem to recover the optimal input state.