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Revisiting Lieb-Thirring Inequalities

The moment inequalities due to Lieb and Thirring are effective tools in the operator theory. Especially the one for the sum of the (negative) eigenvalues of a Schrödinger operator, since, by duality, it is equivalent to a lower bound for the kinetic energy of Fermions, which is exactly of the right semi-classical Thomas–Fermi type.

Based on ideas of Rumin, we show a novel approach of proving the Lieb-Thirring inequalities for the operator $H = |p|^k - U$ with arbitrary $k > 0$ in any dimension d. The obtained constants are improvements of currently known results in all cases, in particular, for $k = 2$.

The other advantage is that the derived factors relating our inequality to semiclassical ones, that is, the quotient of our constants divided by the semi-classical guess, are uniformly bounded for all k and d by e.

We also estimate number of negative eigenvalues for the operator H with dimension $d > k$. Factoring out the semiclassical estimate on the number of bound states yields a uniformly bounded estimate converging to e^2 for large dimensions. These results work for all k and do not use an extension of the bounds to operator-valued potentials and the induction in the dimension trick of Laptev and Weidl, which works only for $k = 2$.

This seems to be the first time that one can prove universal bounds without using some type of induction in the dimension argument. However, for $k = 2$ one can do this and we get bounds which improving the bounds for small values of d in this case.