TOSHIMITSU TAKAESU, Gunma University

Ground States of Quantum Electrodynamics with Cutoffs

We investigate a system of a Dirac field coupled to a quantized radiation field in the Coulomb gauge. The Hilbert space for the system is defined by $\mathcal{F}_{\mathrm{QED}} = \mathcal{F}_{\mathrm{Dirac}} \otimes \mathcal{F}_{\mathrm{rad}}$ where $\mathcal{F}_{\mathrm{Dirac}}$ is the fermion Fock space over $L^2(\mathbb{R}^3_{\mathbf{p}}; \mathbb{C}^4)$ and $\mathcal{F}_{\mathrm{rad}}$ is the boson Fock space over $L^2(\mathbb{R}^3_{\mathbf{k}} \times \{1,2\})$. The total Hamiltonian is defined by

$$H_{\mathrm{QED}} = H_{\mathrm{Dirac}} \otimes I + I \otimes H_{\mathrm{rad}} + \kappa_{\mathrm{I}} \sum_{j=1}^{3} \int_{\mathbb{R}^3} \chi_{\mathrm{I}}(\mathbf{x}) \left(\psi^*(\mathbf{x}) \alpha^j \psi(\mathbf{x}) \otimes A^j(\mathbf{x}) \right) d\mathbf{x}$$
$$+ \kappa_{\mathrm{II}} \int_{\mathbb{R}^6} \frac{\chi_{\mathrm{II}}(\mathbf{x}) \chi_{\mathrm{II}}(\mathbf{y})}{|\mathbf{x} - \mathbf{y}|} \left(\psi^*(\mathbf{x}) \psi(\mathbf{x}) \psi^*(\mathbf{y}) \psi(\mathbf{y}) \otimes I \right) d\mathbf{x} d\mathbf{y},$$

on the Hilbert space. Here H_{Dirac} and H_{rad} denote the field energy Hamiltonians, $\psi(\mathbf{x}) = (\psi^{\,l}(\mathbf{x}))_{l=1}^4$ and $\mathbf{A}(\mathbf{x}) = (A^j(\mathbf{x}))_{j=1}^3$ denote the field operators with ultraviolet cut-offs, and $(\alpha^j)_{j=1}^3$ denote the Dirac matrices. The total Hamiltonian is self-adjoint and bounded from below. We assume spatially localized conditions and momentum regularity conditions. Then it is proven that the total Hamiltonian has a ground state for all values of coupling constants. In particular, its multiplicity is finite.