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Long-range scattering theory for discrete Schrödinger operators

In this talk, we consider discrete Schrödinger operators $H = H_0 + V$ on periodic lattices including the square lattice $\mathbb{Z}^d$ and the hexagonal lattice. We prove that we can construct a long-range scattering theory for a pair of H_0 and H if the perturbation V is a long-range potential. More precisely, we construct time-independent (or Isozaki-Kitada) modifiers $W^{\pm}(\Gamma) = \text{s-lim}_{t\to\pm\infty} e^{itH} J e^{-itH_0} E_{H_0}(\Gamma)$, where Γ is any open set of $\sigma(H_0)$ away from the threshold energies, and prove that they are asymptotically complete. The above modifiers are constructed from a solution of the corresponding eikonal equation on the outgoing and incoming regions of $T^*\mathbb{T}^d$. The proof is analogous to that in the paper by Isozaki and Kitada in 1985; we use the stationary phase method and the Enss method for the proof of the existence and the completeness of $W^{\pm}(\Gamma)$, respectively. The proof for the hexagonal lattice is more complicated, because we need the diagonalization of H_0 and additional argument due to the corresponding Hilbert space $\ell^2(\mathbb{Z}^2;\mathbb{C}^2)$.