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Topological Levinson's theorem for inverse square potentials with infinitely many bound states

Levinson's theorem is a surprising result in quantum scattering theory, which is originally established by N. Levinson for a spherically symmetric potential in 1949. It is a relation between an expression coming from the scattering part of a quantum system and the number of bound states of that system. In 2007, J. Kellendonk and S. Richard gave a topological interpretation of this relation as an index theorem by using K-theory for C*-algebras. More precisely, if the number of bound states are finite, then the Møller wave operators are Fredholm and an index theorem is proved for them via a Toeplitz extension of $C(\mathbb{T})$ by the set of compact operators.

The common feature of these investigations is that the number of bound states of the underlying system is finite, but what happens if infinitely many bound states are involved? In this talk, we consider a model consisting in Schrödinger operators $H_{m,\kappa}$ on a half-line with a potential of the form r^{-2}. Depending on the parameters (m,κ), the wave operators are either Fredholm, semi-Fredholm or almost-periodic operators. We introduce new algebraic frameworks and establish meaningful relations for each cases, which correspond to the topological Levinson's theorem for this model. The resulting relations are concrete realizations of several famous index theorems, e.g. Atiyah's L^2-index theorem.

This talk is based on a joint work with S. Richard.