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*On the eigenvalue embedded in the spectral bands of Schrödinger operators on carbon nanotubes with impurities*

In this talk, we discuss the spectra of Schrödinger operators on carbon nanotubes with impurities from the point of view of the theory of quantum graphs. In the case of carbon nanotubes without impurities, it is known that the spectrum has the band-gap structure, namely, the spectrum consists of infinitely many closed interval (spectral bands) and the flat bands (the set of eigenvalues with infinite multiplicities). In this talk, we give a finite number of impurities expressed as the  $\delta$  vertex conditions to the operator. As a result, we obtain additional eigenvalues embedded in the spectral bands (not in the spectral gaps!). Furthermore, we have an estimate from below of the number of embedded eigenvalues in each spectral bands for a suitable strength of  $\delta$  vertex conditions.

In this talk, we consider the case where impurities are symmetric with respect to  $z$ -axis and rotation. Due to the rotational symmetry, we obtain a unitary equivalence between our operator and the direct sum of a finite number of Schrödinger operators on the degenerate carbon nanotube. Furthermore, we utilize the space-symmetry on  $z$ -axis and decompose those operators as the direct sum of the reduced operators on half size degenerate carbon nanotube with the Dirichlet and Neumann boundary condition. After those decomposition, we examine the estimate from below of the number of eigenvalues in the spectral gaps of each reduced operator. Finally, we show that those eigenvalues are embedded in the spectral bands of other reduced operators.