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An exact power series representation of the Baker-Campbell-Hausdorff formula

The Baker-Campbell-Hausdorff formula is well known and given by $Z = \log(e^X e^Y) = X + Y + \frac{1}{2}[X, Y] + \frac{1}{12}[X, [X, Y] + \frac{1}{12}[Y, [Y, X]] + \dots$, where it is not obvious what the dots represent. Considering the symmetric form of this formula, namely $S(A, B) = \log(e^{A/2} e^B e^{A/2})$, we find an exact power series representation in the matrix B . We find closed form A -dependent coefficients in the form of hyperbolic functions for all orders of B . Each of these coefficients represent an infinite number of terms in the original expansion, making truncation of the series much more controllable for small B but arbitrary A .