Dynamical Systems
Systèmes dynamiques

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PIERRE BERGER, CNRS-Université Paris 13

Emergence of non-ergodic, conservative dynamics

The Birkhoff ergodic theorem states that given an ergodic probability measure μ, for μ-almost every point x, the Birkhoff average:

$$S_k(x) := \frac{1}{k}\sum_{i=1}^{k} \delta_{f^i(x)}$$

converges to μ. For many differentiables maps f, the Birkhoff average pushes forward the Lebesgue measure to a finite number of ergodic probability measures. On the other hand, Newhouse showed in the 70's that a locally generic diffeommorphism may display infinitely many attractors with very different ergodic behavior. I showed that Newhouse phenomena is furthermore locally typical in the sense of Kolmogorov. To describe the complexity of such dynamics, I introduced the following:

Definition: The **Emergence** $\mathcal{E}(\epsilon)$ at scale $\epsilon > 0$ of a system is the minimal number N of probability measures $(\mu_i)_{1\leq i\leq N}$ necessarily so that the Birkhoff average satisfies:

$$\limsup_{k\to\infty} \int_M d_{W_1}(S_k(x), \{\mu_i : 1 \leq i \leq N\}) dLeb < \epsilon \ ,$$

where d_{W_1} is the 1-Wasserstein metric on the space of probability measures of M.

Conjecture (B. 2017): In many categories of dynamical systems, a typical dynamics displays a super polynomial emergence: for every $k > 0$, $\lim_{\epsilon\to 0} \mathcal{E}(\epsilon) \cdot \epsilon^k \to \infty$.

We will present recent developments on this program, and its analog with the theory of systems of positive entropy.

In a work in progress with Jairo Bochi, we showed an analog of the variational principle of the entropy for the concept of emergence. With Turaev, we showed furthermore that in the open set of symplectomorphisms with a totally elliptic periodic point, a typical diffeomorphism in the sense of Kolmogorov (i.e. Leb. a.e. map in a generic family) displays a maximal, super exponential emergence.

AARON BROWN, University of Chicago

Finiteness of actions by higher-rank lattices

For $n \geq 3$, consider a lattice subgroup Γ of $\mathrm{SL}(n, \mathbb{R})$ and a smooth action of Γ on a compact manifold M. With D. Fisher, S. Hurtado, and D. W. Morris, we show that all such actions are finite when $\dim(M) \leq n - 2$. This extends previous results with D. Fisher and S. Hurtado on actions of cocompact lattices. I will give some history and motivation for the problem and briefly explain some tools used in the proof.

MARK DEMERS, Fairfield University

Measure of Maximal Entropy for Finite Horizon Sinai Billiards

While the existence and properties of the SRB measure for the billiard map associated with a periodic Lorentz gas are well understood, there are few results regarding other types of measures for dispersing billiards. We begin by proposing a naive definition of topological entropy for the billiard map, and show that it is equivalent to several classical definitions. We then prove a variational principle for the topological entropy and proceed to construct a measure which achieves the maximum. This measure is Bernoulli and positive on open sets. An essential ingredient is a proof of the absolute continuity of the unstable foliation with respect to the measure of maximal entropy. This is joint work with Viviane Baladi.

ANDREY GOGOLEV, Ohio State University
New partially hyperbolic diffeomorphism by surgery.

I will overview recent progress on examples of partially hyperbolic diffeomorphisms. In particular I will explain new higher dimensional examples constructed by surgery. The latter is joint work with F. Rodriguez Hertz.

ADAM KANIGOWSKI,
Kakutani equivalence of unipotent flows

We study Kakutani equivalence of unipotent flows acting on compact quotients of semisimple Lie groups.For every such flow we compute the Kakutani invariant of M. Ratner, the value of which being explicitly given by the Jordan block structure of the unipotent element generating the flow. Moreover we classify all standard (loosely bernoulli with zero entropy) unipotent flows.

KATHRYN LINDSEY, Boston College
Galois conjugates of growth rates of PCF generalized beta-transformations

The exponential of the topological entropy of any postcritically finite generalized beta-map on an interval is an algebraic integer. Bill Thurston plotted all the Galois conjugates of a large set of "tent maps" (which are a type of generalized beta-map); the visually stunning resulting image showed a set with a rich and complicated geometric structure. A similar set exists for other types of generalized beta-maps. What can we prove about the structure of these sets? I will discuss the existence of "holes" in these sets and the continuity/dependence of the Galois conjugates on the topological entropy. In particular, I will focus on the "persistence" of conjugates inside the unit disk as the entropy increases. This talk is based on joint research with H. Bray, D. Davis and C. Wu.