ICMP Contributed Talks 3
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JOÃO L. COSTA, Lisbon University Institute - ISCTE/CAMGSD

Cosmic no-hair in spherically symmetric black hole spacetimes

We analyze in detail the geometry and dynamics of the cosmological region arising in spherically symmetric black hole solutions of the Einstein-Maxwell-scalar field system with a positive cosmological constant. More precisely, we solve, for such a system, a characteristic initial value problem with data emulating a dynamic cosmological horizon. Our assumptions are fairly weak, in that we only assume that the data approaches that of a subextremal Reissner-Nordström-de Sitter black hole, without imposing any rate of decay. We then show that the radius (of symmetry) blows up along any null ray parallel to the cosmological horizon, in such a way that $r = +\infty$ is, in an appropriate sense, a spacelike hypersurface. We also prove a version of the Cosmic No-Hair Conjecture by showing that in the past of any causal curve reaching infinity both the metric and the Riemann curvature tensor asymptote those of a de Sitter spacetime. Finally, we discuss conditions under which all the previous results can be globalized.

MARC DE MONTIGNY, University of Alberta

The Dirac oscillator in a non-commutative cosmic string spacetime

We examine the non-inertial effects on a Dirac oscillator of a rotating frame in a cosmic string spacetime with non-commutative geometry. We discuss the bound states solutions of the Dirac equation and the corresponding energies. We observe, in some limits, a coupling between the angular momentum and the angular velocity, and between the spin and the angular velocity. For the non-relativistic limit, we notice the existence of a degeneracy frequency given a specific relation between the oscillator frequency and the rotating frequency, for which the energy levels become degenerate.

DAVID FAJMAN, University of Vienna

Nonvacuum stability of the Milne model

The Milne model is the only cosmological vacuum solution to Einstein's equations (with vanishing cosmological constant), for which nonlinear (future-) stability is proved, due to the work of Andersson-Moncrief. We present a first generalisation of this result to the nonvacuum case, namely to the Einstein-Vlasov system. This system models spacetimes containing ensembles of self-gravitating, collisionless particles. We, in particular, introduce a new technique to combine earlier approaches to control the energy-momentum tensor of massive collisionless matter in cosmological spacetimes with a physically motivated energy estimate that is necessary to establish sufficient decay properties of the matter fields. This is joint work with Lars Andersson.

SUMARNA HAROON, National University of Sciences and Technology, Islamabad

The Effects of Running Gravitational Coupling On Rotating Black Holes

Motivated by the functional form of the gravitational coupling previously investigated in the context of infra-red limit of asymptotic safe gravity theory, I investigate the consequences of a running gravitational coupling for the properties of rotating black holes. Apart from the changes induced in the space-time structure of such black holes, there are also implications for the Penrose process and geodetic precession. In this approach, a new parameter $\tilde{\xi}$ is present in this solution, and I describe the Killing horizon, event horizon and singularity of the resultant metric. Particle geodesics, both null and timelike, are explored in the equatorial plane, and the effective potential is computed and graphically analyzed for different values $\tilde{\xi}$. The ergosphere increases as $\tilde{\xi}$ increases, and energy extraction via the Penrose process is described. For a given value of the spin parameter, the numerical results suggest that the efficiency of Penrose process is greater in asymptotically safe gravity than in the Kerr Black Hole. Finally a brief discussion on the Lense-Thirring frequency is also done.

IGOR KHAVKINE, Czech Academy of Sciences

Linear local gauge-invariant observables on spacetimes of sub-maximal symmetry

The Killing operator $K_{ab}[v] = \nabla_a v_b + \nabla_b v_a$ is the generator of gauge symmetries (linearized diffeomorphisms) $h_{ab} \mapsto h_{ab} + K_{ab}[v]$ in linearized gravity. A linear local gauge-invariant observable is a differential operator $I[h]$ such that $I[K[v]] = 0$ for any gauge parameter field v_a. A set $\{I_i[h]\}$ of such observables is complete if the simultaneous conditions $I_i[h] = 0$ are sufficient to conclude that the argument is a pure gauge mode, $h_{ab} = K_{ab}[v]$. The explicit knowledge of a complete set of local gauge invariant observables has multiple applications from the points of view of both physics and geometry, whenever a precise separation of physical and gauge degrees of freedom is required. Surprisingly, until very recently, such complete sets have been known explicitly only on spacetimes of maximal symmetry (Minkowski or (anti-)de Sitter). I will discuss recent progress that has allowed an explicit construction of complete sets of local gauge invariant observables on backgrounds of sub-maximal symmetry, most notably on cosmological (FLRW) and black hole (Schwarzschild and Kerr) spacetimes.

PAUL KLINGER, University of Vienna

Non-singular stationary spacetimes with negative cosmological constant

We construct of a wide variety of stationary spacetimes with negative cosmological constant. These include families of solutions with various matter fields (including Maxwell, Yang-Mills, dilaton,...), black hole solutions, and boson stars. The solutions are parameterized by freely prescribable functions specifying the asymptotics of the metric and possible matter fields at conformal infinity. The construction uses an implicit function argument around "non-degenerate" vacuum solutions (defined by requiring an operator associated with the linearization of the equations to be an isomorphism). As the Anti-de Sitter and Schwarzschild AdS spacetimes fulfill this condition, we obtain infinite dimensional families of solutions close to these, including a family of solutions with the usual AdS conformal structure at conformal infinity.

This is joint work with Piotr Chrusciel and Erwann Delay.

TIM-TORBEN PAETZ, University of Vienna

On the smoothness of the critical sets of the cylinder at spatial infinity in vacuum spacetimes

According to Penrose, a spacetime is "asymptotically flat" if it admits a smooth conformal compactification at infinity. This raises the question to what extent Einstein's field equations are compatible with his definition, which has been extensively discussed in the literature with a wide range of opinions. This underlines that the restrictions on the physical data need to be understood which allow for a smooth conformal compactification at infinity in order to give a definite answer whether Penrose's proposal captures sufficiently good models for the physical situations of interest.

A convenient setting to analyze this problem is an appropriately conformally rescaled spacetime where spatial infinity is blown up to a cylinder. It is expected that non-smoothness of null infinity is related to non-smoothness of the critical sets where the cylinder "touches" null infinity. We will consider the appearance of logarithmic terms at these critical sets starting from an "asymptotic initial value problem", where data are prescribed at null infinity.

DANIEL SIEMSSEN, University of Wuppertal

A geometric Weyl quantization and asymptotics of natural operators on pseudo-Riemannian manifolds

On flat space $\mathbb{R}^d$, several distinguished quantizations are available. One can argue that the Weyl quantization is the most natural choice and that it has the best properties (e.g., symplectic covariance, real symbols correspond to Hermitian operators). On a generic manifold, there is no distinguished quantization, and a quantization is typically defined chart-wise.

In this talk I will present a quantization that, we believe, has the best properties for studying natural operators on pseudo-Riemannian manifolds. I will also describe the application of this quantization to the computation of the asymptotics of the heat semigroup $\mathrm{e}^{t\Delta}$ and the Green operator $(\Delta + m^2)^{-1}$, as well as their Lorentzian counterparts, the proper time dynamics $\mathrm{e}^{\mathrm{i}s\Box}$ and the Feynman propagator $(\Box - \mathrm{i}0)^{-1}$.

Joint work with J. Dereziński and A. Latosiński.

GERARDO BARRERA VARGAS, University of Alberta

On the switch-type convergence for stochastic differential equations

Consider an ordinary differential equation with a fixed point that is a hyperbolic global attractor. Assume that the fixed point is the origin. Under general conditions, at times goes by any solution of this equation approaches the fixed point exponentially fast. Now add a small random perturbation to this equation. It is well known that, again under very general conditions, as times goes by the solution of this stochastic equation converges to an equilibrium distribution that is well approximated by a Gaussian random variable of variance proportional to the strength of the perturbation. General theory of stochastic processes allows to show that this convergence, for each fixed perturbation, is again exponentially fast. We show that the convergence is actually abrupt: in a time windows of small size compared to the natural time scale of of the process, the distance to equilibrium drops from its maximal possible value to near zero, and only after this time window the convergence is exponentially fast. This is what is known as the cut-off phenomenon in the context of Markov chain of increasing complexity. Under a proper time scaling, we are able to prove convergence of the distance to equilibrium to a universal function, a fact known as profile cut-off. Moreover, when the attractor is not hyperbolic then we are able to prove that the cut-off phenomena does not appears. Joint work with Milton Jara.

ATUL KUMAR VERMA, Department of Mathematics, Indian Institute of Technology Ropar, India-140001.

Modelling of Stochastic Transport Problems using Multi-Channel Exclusion Processes

Many natural systems exhibit complex behavior under stationary state when either driven by some external field or self driven. Such driven diffusive systems reveal very rich nonequilibrium phenomena in physics, chemistry and biology. In order to analyze the collective properties of these driven stochastic transport problems, totally asymmetrically simple exclusion process (TASEP) model is found to be a paradigmatic model to study such problems in the last decade. TASEP is comprised of particles performing biased hopping with a uniform rate in a preferred direction along a 1D lattice. The particles obey certain preassigned rules under hard-core exclusion principle, due to which a lattice site cannot have more than one particle.

In this talk, I will begin with some beautiful theoretical results on single and two-channel exclusion process followed by results on coupled as well as uncoupled systems with or without a nonconserving dynamics known as Langmuir Kinetics. Additionally, to mimic some stochastic transport problems more realistically, we extend two channel system into three- channel systems in the presence of the attachment-detachment process with open boundary conditions. To understand the collective system dynamics, we derive various phase diagrams and density profiles using mean-field theory and singular perturbation technique, for various parameters. Monte-Carlo simulations are carried out for verifying our theoretical findings, which are in good agreement with theoretical findings.

ANNA VERSHYNINA, University of Houston

Recovery map stability for the Data Processing Inequality

The Data Processing Inequality (DPI) states that the Umegaki relative entropy is non-increasing under the action of completely positive trace preserving (CPTP) maps. A theorem of Petz says that there is equality in DPI if and only if both states can be recovered perfectly after passing through a CPTP map. Such recovery map is called Petz recovery map. A standing problem is to obtain a proper lower bound on the difference between relative entropies of input and output states. We provide a quantitative version of Petz's theorem, where the lower bound contains a distance between a state and its Petz's recovered state. The novelty of the result is that for the first time the distance measure contains the original Petz recovery map. Moreover, I will present stability bounds for the quasi-relative entropies defined in terms of an operator monotone decreasing functions, which also includes the distance measure of the state and its Petz's recovered state. The present treatment is developed in the context of finite dimensional von Neumann algebras where the results are already non-trivial and of interest in quantum information theory. (*Joint work with Eric A. Carlen)*

DMITRY VOROTNIKOV, Universidade de Coimbra

Nonlinear Fokker-Planck equations with nonlocalities and reaction as gradient flows

We interpret a class of nonlinear Fokker-Planck equations [1] with reaction as gradient flows over the spaces of Radon and probability measures equipped with the recently introduced Hellinger-Kantorovich distance [2-4] and the spherical Hellinger-Kantorovich distance [5], resp. The latter ones have nonlocal terms which appear as Lagrange multipliers due to conservation of the mass. We prove new isoperimetric-type functional inequalities, which allow us to control the relative entropy by its production. This yields exponential convergence of the trajectories to the equilibrium.

Based on a joint work with S. Kondratyev.

1. T. D. Frank, Nonlinear Fokker-Planck equations: Fundamentals and applications, Springer, 2005.

2. S. Kondratyev, L. Monsaingeon, D. Vorotnikov, A new optimal transport distance on the space of finite Radon measures, Adv. Differential Equations 21 (2016) 1117-1164.

3. M. Liero, A. Mielke, G. Savare, Optimal entropy-transport problems and a new Hellinger-Kantorovich distance between positive measures, to appear in Invent. Math.

4. L. Chizat et al. An Interpolating Distance Between Optimal Transport and Fisher-Rao Metrics, to appear in Found. Comp. Math.

5. A. Mielke and V. Laschos. Geometric properties of cones with applications on the Hellinger-Kantorovich space, and a new distance on the space of probability measures, preprint.