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ANDREA AGAZZI, Duke University
Large deviations theory for chemical reaction networks

At the microscopic level, the dynamics of arbitrary networks of chemical reactions can be modeled as jump Markov processes whose sample paths converge, in the limit of large number of molecules, to the solutions of a set of algebraic ordinary differential equations. Fluctuations around these asymptotic trajectories and the corresponding phase transitions can in principle be studied through large deviations theory in path space, also called Wentzell-Freidlin (W-F) theory. However, the specific form of the jump rates for this family of processes does not satisfy the standard regularity assumptions imposed by such theory, and weaker conditions need to be developed to deal with the framework at hand.

Using tools of Lyapunov stability theory we design sufficient conditions for the applicability of large deviations estimates to the asymptotics of the Markov process at hand. We then translate such conditions in terms of the topological structure of the chemical reaction network. This ultimately allows to define a large class of chemical reaction systems to which the estimates of interest can automatically be applied.

TOSHIAKI AIDA, Okayama University
Replica Analysis of the Performance of Compressed Sensing for Image Processing

Compressed sensing is one of the most effective signal processing methods based on the sparse representation of inferred data, in which dictionary matrices play an essential role as a set of overcomplete basis vectors. They are learned from data by feature extraction methods such as K-SVD ones. Therefore, in general, it requires a considerable amount of computational cost to construct a dictionary matrix.

In this work, we analytically evaluate the performance of compressed sensing for image processing, utilizing the replica method of statistical physics. For this purpose, we have derived the expression of the probability distribution followed by a dictionary matrix for image processing, assuming that grey scale images are generated by the Gaussian model [1]. Also, we have adopted the framework of replica analysis by Kabashima et. al. to analytically evaluate the performance of compressed sensing based on Lp-norm minimization [2].

Our analysis has made clear, for example, the scaling relation between the complexity of images and the optimal size of dictionary matrices, which has been addressed not in an analytical way but only in a numerical one.

[1] Y. Ashida and T. Aida, “Probability Distribution of an Image Dictionary for Compressed Sensing,” Proceedings of 2016 16th International Conference on Control, Automation and Systems, pp. 1377-1380, 2016.

[2] Y. Kabashima, T. Wadayama and T. Tanaka, “A typical reconstruction limit for compressed sensing based on Lp-norm minimization,” Journal of Statistical Mechanics: Theory and Experiment, (2009) L09003.

JOHANNES ALT, IST Austria
Correlated random matrices: Dyson equation and edge universality

The eigenvalue density of many large Hermitian random matrices is well-approximated by a deterministic measure on $\mathbb{R}$, the *self-consistent density of states*. In the case of an $N \times N$ random matrix with nontrivial expectations of its entries or a nontrivial correlation among them, this measure is obtained from the matrix Dyson equation on $N \times N$ matrices. The matrix Dyson equation generalizes scalar- or vector-valued Dyson equations that have been studied previously. In this talk, we will show that the self-consistent density of states is real-analytic apart from finitely many square root edges and cubic root cusps. We will also explain how detailed information about these singularities can be used to prove Tracy-Widom fluctuation for the eigenvalues close to the square root edges of the associated self-consistent density of states. This is joint work with László Erdős, Torben Krüger and Dominik Schröder.

JÉRÉMIE BOUTTIER, CEA Saclay and ENS de Lyon
Edge behavior of the periodic and the free boundary Schur processes

The Schur process is in some sense a discrete analogue of a random matrix. Their edge behavior are known to be in the same universality class, described by the Airy kernel and the Tracy-Widom distribution. In this talk we consider two variants of the Schur process: the periodic case introduced by Borodin, and the "free boundary" case recently introduced by us. We are able to compute their correlation functions in a unified manner using the machinery of free fermions. We then investigate the edge asymptotic behavior and show it corresponds to two nontrivial deformations of the Airy kernel and of the Tracy-Widom distribution. Based on joint work with Dan Betea, Peter Nejjar and Mirjana Vuletić.

FABIO DEELAN CUNDEN, University College Dublin
Moments of random matrices and hypergeometric orthogonal polynomials

We establish a new connection between moments of $n \times n$ random matrices X_n and hypergeometric orthogonal polynomials. Specifically, we consider moments $\mathbb{E}\mathrm{Tr} X_n^{-s}$ as a function of the complex variable $s \in \mathbb{C}$, whose analytic structure we describe completely. We discover several remarkable features, including a reflection symmetry (or functional equation), zeros on a critical line in the complex plane, and orthogonality relations. An application of the theory resolves part of an integrality conjecture of Cunden et al. [F. D. Cunden, F. Mezzadri, N. J. Simm and P. Vivo, J. Math. Phys. 57 (2016)] on the time-delay matrix of chaotic cavities. In each of the classical ensembles of random matrix theory (Gaussian, Laguerre, Jacobi) we characterise the moments in terms of the Askey scheme of hypergeometric orthogonal polynomials. We also calculate the leading order $n \to \infty$ asymptotics of the moments and discuss their symmetries and zeroes. We discuss aspects of these phenomena beyond the random matrix setting, including the Mellin transform of products and Wronskians of pairs of classical orthogonal polynomials. When the random matrix model has orthogonal or symplectic symmetry, we obtain a new duality formula relating their moments to hypergeometric orthogonal polynomials.

This is joint work with Francesco Mezzadri, Neil O'Connell and Nick Simm.

JI OON LEE, Korea Advanced Institute of Science and Technology
Free Energy of Spherical Sherrington-Kirkpatrick Model

Spherical Sherrington-Kirkpatrick (SSK) model is an example of disordered systems known as spin glasses. The free energy of 2-spin SSK, which is a random variable, can be considered as a finite temperature version of the largest eigenvalue of a random symmetric matrix. We find the critical temperature and prove the limiting free energy for all non-critical temperature. We also show that the law of the fluctuation of the free energy converges to the Gaussian distribution when the temperature is above the critical temperature, and to the GOE Tracy-Widom distribution when the temperature is below the critical temperature. The result is universal, and the analysis is applicable to various models including SSK with ferromagnetic interaction and bipartite SSK model. This talk is based on joint works with Jinho Baik.

ZHIPENG LIU, University of Kansas
Multi-point distribution of periodic TASEP

The height fluctuations of the models in the KPZ class are expected to converge to a universal process. The spatial process at equal time is known to converge to the Airy process or its variations. However, the temporal process, or more generally the two-dimensional space-time fluctuation field, is less well understood. We consider this question for the periodic TASEP (totally asymmetric simple exclusion process). For a general initial condition, we evaluate the multi-time and multi-location distribution explicitly in terms of a multiple integral involving a Fredholm determinant. With some assumptions on the initial condition, we evaluate the large time limit of this multi-point distribution in the so-called relaxation time scale.

ASAD LODHIA, University of Michigan
Covariance for LSS of Wigner Matrices with Few Moments

We examine the covariance kernel for linear spectral statistics of Wigner matrices when the underlying distribution has a fourth moment but does not have a variance, and derive a simplified expression. This is joint work with Anna Maltsev.

ANNA MALTSEV, Queen Mary University of London
Localization and landscape functions on quantum graphs

I will discuss localization and other properties of eigenfunctions of the Schrödinger operator on quantum graphs. The motivation is to understand how graph structure impacts eigenfunction behavior. I will present two estimates based on the Agmon method to show that a tree structure aids the exponential decay at energies below the essential spectrum. I will furthermore present adaptations of the landscape function approach, well-established for $\mathbb{R}^n$, to quantum graphs and its limitations. In our context, a "landscape function" $\Upsilon(x)$ is a function that controls the localization properties of normalized eigenfunctions $\psi(x)$ through a pointwise inequality of the form $|\psi(x)| \leq \Upsilon(x)$. The connectedness of a graph can present a barrier to the existence of universal landscape functions in the high-energy régime, as we demonstrate with simple examples. However, at low and moderate energies landscape functions can be made explicit. This talk is based on joint work with Evans Harrell.

GIOVANNA MARCELLI, Universität Tübingen
Spin conductance and spin conductivity in topological insulators: analysis of Kubo-like terms

We investigate spin transport in 2-dimensional insulators, with the long-term goal of establishing whether any of the transport coefficients corresponds to the Fu-Kane-Mele index which characterizes $2d$ time-reversal-symmetric topological insulators. Inspired by the Kubo theory of charge transport, and by using a proper definition of the spin current operator [Phys. Rev. Lett. 96, 076604 (2006)], we define the Kubo-like spin conductance $G_K^{s_z}$ and spin conductivity $\sigma_K^{s_z}$. We prove that for any gapped, periodic, near-sighted discrete Hamiltonian, the above quantities are mathematically well-defined and the equality $G_K^{s_z} = \sigma_K^{s_z}$ holds true. Moreover, we argue that the physically relevant condition to obtain the equality above is the vanishing of the mesoscopic average of the spin-torque response, which holds true under our hypotheses on the Hamiltonian operator. This vanishing condition might be relevant in view of further extensions of the result, e.g. to ergodic random discrete Hamiltonians or to Schrödinger operators on the continuum. A central role in the proof is played by the trace per unit volume and by two generalizations of the trace, the principal value trace and its directional version. This talk is based on joint work with Gianluca Panati (La Sapienza, Roma) and Clément Tauber (ETH, Zürich).

YURIY NEMISH, IST Austria
Local laws for polynomials of Wigner matrices

We consider general self-adjoint polynomials in several independent random matrices whose entries are centered and have constant variance. Under some numerically checkable conditions, we establish the optimal local law, i.e., we show that the empirical spectral distribution on scales just above the eigenvalue spacing follows the global density of states which is determined by free probability theory. First, we give a brief introduction to the linearization technique that allows to transform the polynomial model into a linear one, which has simpler correlation structure but higher dimension. After that we show that the local law holds up to the optimal scale for the generalized resolvent of the linearized model, which also yields the local law for polynomials.

This is a joint work with Laszlo Erdös and Torben Krüger.

DIMITRI PETRITIS, Université de Rennes 1, France
Heavy-tailed random walks in the critical regime

Many important phenomena in Physics are governed, modelled, or controlled by heavy-tailed processes (eg. Lévy flights in atom cooling, distributions of thermodynamic functions of systems in random environments, etc.). Heavy-tailed processes are also ubiquitous in other disciplines (extreme value theory in mathematics, traffic modelling or small world effects in networks, wealth distribution in social economy, reliability theory, population dynamics, etc.).

We shall review sharp classification results [1, 3, 2] of the asymptotic behaviour of random walks whose increments are heavy-tailed; these results are obtained by use of Lyapunov functions methods transforming the studied Markov processes into semi-martingales.

References

[1] Vladimir Belitsky, Mikhail Menshikov, Dimitri Petritis, and Marina Vachkovskaia. Random dynamical systems with systematic drift competing with heavy-tailed randomness. *Markov Proc. Rel. Fields*, 22:629–652, 2016.

[2] Nicholas Georgiou, Mikhail Menshikov, Dimitri Petritis, and Andrew Wade. Lamperti's problem for heavy-tailed random walks. in preparation, 2018.

[3] Mikhail V. Menshikov, Dimitri Petritis, and Andrew R. Wade. Heavy-tailed random walks on complexes of half-lines. *Journal of Theoretical Probability*, pages 1–41, 2017.

MARTIN ZIRNBAUER, Institute for Theoretical Physics, University of Cologne, Germany

The integer quantum Hall plateau transition is a current algebra after all

The scaling behavior near the transition between plateaus of the Integer Quantum Hall Effect (IQHE) has traditionally been interpreted on the basis of a two-parameter renormalization group flow conjectured from Pruisken's non-linear sigma model. Yet, the conformal field theory (CFT) describing the critical point remained elusive, and only fragments of a quantitative analytical understanding existed up to now. Here we carry out a detailed study of the current-current correlation function for the conductivity tensor, initially in the Chalker-Coddington network model for the IQHE plateau transition and then in its exact reformulation as a supersymmetric vertex model. We take the continuum limit of the non-local conductivity response function at criticality and thus identify a non-Abelian current algebra at level $n = 4$. By proposing precise lattice expressions for the CFT primary fields we predict the multifractal scaling exponents of critical wave intensities to be $\Delta_q = q(1-q)/4$.