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VINCENT BEAUD, Technical University Munich
Bounds on the entanglement entropy of droplet states in the XXZ spin chain

We consider a class of one-dimensional quantum spin systems on the finite lattice $\Lambda \subset \mathbb{Z}$, related to the XXZ spin chain in its Ising phase. It includes in particular the so-called droplet Hamiltonian. Remarkably, the attractive, nearest-neighbor interaction implies the existence of thresholds in the spectrum of the corresponding Hamiltonians: clustering of spins with the same orientation is energetically favorable and a lowest band may be identified, the "droplet band". It should be emphasized that, generically, droplet states are neither Gaussian, nor ground states of our system. The entanglement entropy of these energetically low-lying states over a bipartition $\Lambda = B \cup B^c$ is investigated and proven to satisfy a logarithmic bound in terms of $\min\{n, |B|, |B^c|\}$, where n denotes the maximal number of down spins in the considered state. Upon addition of any (positive) random potential the bound becomes uniformly constant on average, thereby establishing an area law. The proof is based on spectral methods: a deterministic bound on the local (many-body integrated) density of states is derived from an energetically motivated Combes–Thomas estimate. (Joint work with S. Warzel)

TRISTAN BENOIST, Institut de Mathématiques de Toulouse
Hypothesis testing of quantum channel unravelings

A quantum channel can always be unraveled to an instrument. This instrument defines then a probability measure describing the repeated measurement of a quantum system. This models experiments in quantum optics. In this talk I will present results obtained with N. Cuneo, V. Jaksic, Y. Pautrat and C.-A. Pillet on the hypothesis testing between two instruments. First I will characterize the identifiable parameters of the instrument showing that the instrument is identifiable up to isomorphism and eventually only on a sub algebra. Then I will explain how the sub multiplicativity of the probability measures defined by the instruments implies that the log of the likelihood ratio between the two tested probability measures converge almost surely. That convergence then implies Stein's Lemma. Hence Neyman-Pearson tests saturate Stein's error exponents. I will then explain how, using the sub additive thermodynamic formalism borrowed from classical statistical mechanics, we show that the limit Rényi's relative entropy between the two probability measures exists and is differentiable for orders in $(0, 1)$. This regularity implies the equality between all Hoeffding's error exponents at fixed rate and both Chernoff's error exponents. Again that shows associated Neyman-Pearson tests saturate these error exponents. I illustrate our results first with physically relevant examples and then with examples illustrating the sharpness of our results. I will give examples of finite and infinite Rényi's relative entropy, non analytic Rényi's relative entropy, non Gaussian central limit for the log likelihood ...

GIACOMO DE PALMA, QMATH, University of Copenhagen, Department of Mathematical Sciences
The Conditional Entropy Power Inequality for Bosonic Quantum Systems

We prove the conditional Entropy Power Inequality for Gaussian quantum systems. This fundamental inequality determines the minimum quantum conditional von Neumann entropy of the output of the beam-splitter or of the squeezing among all the input states where the two inputs are conditionally independent given the memory and have given quantum conditional entropies. We also prove that, for any couple of values of the quantum conditional entropies of the two inputs, the minimum of the quantum conditional entropy of the output given by the conditional Entropy Power Inequality is asymptotically achieved by a suitable sequence of quantum Gaussian input states. Our proof of the conditional Entropy Power Inequality is based on a new Stam inequality for the quantum conditional Fisher information and on the determination of the universal asymptotic behaviour of the quantum conditional entropy under the heat semigroup evolution. The beam-splitter and the squeezing are the central elements of quantum optics, and can model the attenuation, the amplification and the noise of electromagnetic signals. This conditional Entropy Power Inequality will have a strong impact in quantum information and quantum cryptography. Among its many possible applications there is the proof of a new uncertainty relation for the conditional Wehrl entropy.

Based on GdP and Dario Trevisan, Communications in Mathematical Physics (2018), https://doi.org/10.1007/s00220-017-3082-8

FRANK HANSEN, Tohoku University

Quantum entropy derived from first principles

The most fundamental properties of quantum entropy are derived by considering the union of two ensembles. We discuss the limits these properties put on an entropy measure and obtain that they uniquely determine the form of the entropy functional up to normalisation. In particular, the result implies that all other properties of quantum entropy may be derived from these first principles.

Quantum entropy derived from first principles. Journal of Statistical Physics (2016) 165(5):799-808. Doi 10.1007/s10955-016-1651-4.

WATARU ICHINOSE, Shinshu University

Mathematical theory of the Feynman path integrals for quantum continuous measurement of positions and spin

Quantum spin systems of many particles moving in an electric-magnetic field are considered. Let's make measurements simultaneously of both positions of every spin component of all particles and spin directions of all particles continuously during a finite time. Then, taking account of reaction to measurement, the probability amplitudes of particles with spin after this continuous measurement are given by the restricted Feynman path integrals or the weighted Feynman path integrals, as has been proposed by Feynman and Mensky. Our aim in this talk is to prove under some general assumptions about the electric-magnetic field that the weighted Feynman path integrals corresponding to this measurement are well defined mathematically and satisfy the non-self-adjoint Pauli equations for quantum spin systems of many particles.

In our proof a sharp estimate for the L^2-boundedness of the pseudo-differential operators, stated in Zworski's book (2012), plays a key role. When the error of our measurement is infinite, which means no measurement at all, then our weighted Feynman path integrals become equal to the usual Feynman path integrals for spin systems of many particles.

MARCO LETIZIA, University of Waterloo

Quantum fields on causal sets and entaglement entropy

In this talk I will discuss some algebraic properties of two classes of quantum scalar fields living in a Lorentz invariant discrete background described by a causal set. In particular I will show how certain features of these models affect the behavior of the entanglement entropy, its divergences and the emergence of the area-law.

ANGELO LUCIA, QMATH, University of Copenhagen

Locality at the boundary implies gap in the bulk for 2D PEPS

Proving that the parent Hamiltonian of a Projected Entangled Pair State (PEPS) is gapped remains an important open problem. We take a step forward in solving this problem by showing two results: first, we identify a condition on the boundary state of rectangular subregions that is sufficient to prove that the the parent Hamiltonian of the bulk 2D PEPS has a constant gap in the thermodynamic limit; second, we then show that boundary states which are Gibbs state of a local Hamiltonian on the virtual indices with interactions decaying faster than exponentially in the distance satisfy such condition. The proof employs the martingale method of nearly commuting projectors, and exploits a result of Araki on the robustness of one dimensional Gibbs states. Our result provides one of the first rigorous connections between boundary theories and dynamical properties in an interacting many body system.

TADAHIRO MIYAO, Hokkaido University

On the semigroup generated by the renormalized Nelson Hamiltonian

In this talk, I will report my recet results of the renormalized Nelson model at a fixed total momentum. Because the Hamiltonian is defined through an infinite energy renormalization, analysis of this model is known to be difficult. After a reveiw of ealier studies, I will briefly explain that the semigroup is the positivity improving in the Fock representation.

ATMN PATEL, University of Waterloo
An Analytic Approach to Quantum Shannon Theory

We characterize the conditions under which the quantum channel capacities are analytic. If the optimization over input states is restricted to positive operators, the quantum and classical channel capacities are analytic on the set of channels. This proof arises naturally from Danskin's Theorem which characterizes the Gatiéaux differential of an optimization function and a number of standard theorems from complex function theory. We calculate the general power series expansion of the n-shot classical and quantum capacity using standard techniques from functional calculus. This can be used to directly compute the n-shot quantum capacity without any optimization given that the center of the expansion is a degradable channel. As a direct application of computing these derivatives, we develop a necessary and sufficient criterion for n-shot superactivation. It is also shown that the power series expansion can be inverted using Lagrange's Inversion Theorem to recover the optimal input state.

JASON PYE, University of Waterloo
A Universal Training Algorithm for Quantum Deep Learning

Here we introduce a training algorithm for deep quantum neural networks and parametric quantum circuits. The core of the algorithm unifies the phase kickback principle of quantum computing with the backpropagation algorithm of classical machine learning. This, along with one of a number of possible optimisation strategies, can be used to train continuously-parametrised classical and quantum machines. We illustrate several possible applications for this new training algorithm.

ROBIN REUVERS, University of Cambridge
Lower bound on entanglement in subspaces defined by Young diagrams

Eigenvalues of 1-particle reduced density matrices of N-fermion states are upper bounded by $1/N$, resulting in a lower bound on entanglement entropy. In this talk, I discuss the optimal eigenvalue bound for all other subspaces defined by Young diagrams in the Schur–Weyl decomposition of $\otimes^N \mathbb{C}^d$ [1]. I also mention the numerical algorithm that facilitated the search for a proof [2].

[1] Lower bound on entanglement in subspaces defined by Young diagrams, arXiv (from 2 May)

[2] An algorithm to explore entanglement, arXiv:1711.07943

PER VON SOOSTEN, TU Munich
The Localization Transition in the Ultrametric Ensemble

This talk focuses on a Dyson-hierarchical analogue of the power-law random band matrices with Gaussian entries whose variances decay in a suitable ultrametric. The model can be constructed recursively by alternating between averaging independent copies of the matrix and running Dyson Brownian motion and we use this observation to prove the existence of a localization transition. We map out the entire localized regime in terms of Poisson statistics and eigenfunction decay. We also prove random matrix universality and the existence of maximally extended eigenfunctions beyond the mean-field regime. In this regime, the model has a well defined infinite-volume limit for which we prove the Holder-continuity of the spectral measures. This talk is based on joint work with Simone Warzel.