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ABDELMALEK ABDESSELAM, University of Virginia
A proof of weak universality for a non-Gaussian scaling limit with anomalous scaling

Wilson's theory of critical phenomena provides a heuristic yet compelling explanation for the mechanism behind non-Gaussian scaling limits with anomalous scaling dimensions and the associated universality property. Such a scaling limit corresponds to a nontrivial renormalization group fixed point and the basin of attraction featuring in the universality property is the stable manifold of that fixed point. I will review results of joint work with A. Chandra and G. Guadagni where we turned Wilson's ideas into rigorous mathematical theorems. The model we studied is a hierarchical 3D ferromagnet with continuous spins, slightly below the upper critical dimension. The scaling limit is for the joint law of the spin field and its square. While the spin field does not exhibit anomalous scaling, we proved that the square/energy field does, which confirms a prediction made by Wilson in 1972. The (joint) scaling limit is shown to be the same for a large class of (irrelevant) interactions ϕ^6, ϕ^8, etc. However, this universality result which we proved is in the "weak sense", namely, these interactions are assumed to be small. Our result is based on a new rigorous renormalization group method which allows couplings to be space-dependent.

LAURENT BÉTERMIN, University of Copenhagen
Lattice theta function and ionic crystal: a proof of Born's conjecture

In his paper "Über elektrostatische Gitterpotentiale", published in 1921, Max Born asked the following question related to ionic crystals: "How to arrange positive and negative charges on a simple cubic lattice of finite extent so that the electrostatic energy is minimal?". He conjectured that the alternation of charges $+1$ and -1 is optimal distribution of charges.

In this talk, I will explain a connection between the translated lattice theta function and the optimal configuration of charges on a given lattice, when the interaction potential is completely monotone. Thus, a proof of Born's conjecture in any dimension, for orthorhombic lattices, will be given. Finally, we will see that the solution for the triangular lattice exhibits a surprising honeycomb structure. This talk is based on joint works with Hans Knüpfer (University of Heidelberg) and Mircea Petrache (PUC Chile).

CHIARA BOCCATO, IST Austria
Complete Bose-Einstein Condensation in the Gross-Pitaevskii Regime

We consider a gas of N bosons in a box interacting through a potential with scattering length of order N^{-1} (Gross-Pitaevskii limit). Assuming the interaction potential to be sufficiently small, we show that all low-energy states exhibit complete Bose-Einstein condensation. The main novelty of our result is a uniform bound on the number of excited particles, i.e. the condensate depletion.

Joint work with Christian Brennecke, Serena Cenatiempo and Benjamin Schlein.

JOE CHEN, Colgate University
Scaling limit of the exclusion process on resistance spaces

I will describe recent results and challenges on implementing the hydrodynamic program to obtain scaling limits of the weakly asymmetric exclusion process on low-dimensional spaces, namely, those which are bounded with respect to the resistance metric, which include trees, fractals, and random graphs. The key functional tool is an energy inequality for the exclusion process on a finite weighted graph, which leads to a coarse-graining (local ergodic) lemma that enables the passage from the microscopic model to the macroscopic (nonlinear) heat equation.

This is based on joint works with M. Hinz and A. Teplyaev, cf. arXiv:1606.01577, 1702.03376, 1705.10290, and forthcoming papers.

ROBERTO CORTEZ, Universidad de Valparaíso
Uniform propagation of chaos for the spatially homogeneous Boltzmann equation

The spatially homogeneous Boltzmann equation models the evolution of the velocity distribution of a huge number of particles in a gas, subjected to elastic random binary collisions. In this work we study its corresponding finite stochastic N-particle system, and we are interested in the propagation of chaos property: the convergence, as $N \to \infty$ and for each time $t \geq 0$, of the empirical measure of the system towards the solution of the Boltzmann equation. Using recent probabilistic coupling techniques we find, under suitable moments assumptions on the initial distribution, an explicit uniform-in-time propagation of chaos rate of order almost $N^{-1/3}$ in squared 2-Wasserstein distance for the Boltzmann equation in the Maxwell molecules case.

ANDREAS DEUCHERT, Institute of Science and Technology Austria
Bose-Einstein Condensation in a Dilute, Trapped Gas at Positive Temperature

We consider an interacting, dilute Bose gas trapped in a harmonic potential at a positive temperature. The system is analyzed in a combination of a thermodynamic and a Gross-Pitaevskii (GP) limit where the trap frequency ω, the temperature T and the particle number N are related by $N \sim (T/\omega)^3 \to \infty$ while the scattering length is so small that the interaction energy per particle around the center of the trap is of the same order of magnitude as the spectral gap in the trap.

We prove that the difference between the canonical free energy of the interacting gas and the one of the noninteracting system can be obtained by minimizing the GP energy functional. We also prove Bose-Einstein condensation in the following sense: The one-particle density matrix of any approximate minimizer of the canonical free energy functional is to leading order given by the one of the noninteracting gas but with the free condensate wavefunction replaced by the GP minimizer.

This is joint work with Robert Seiringer and Jakob Yngvason

PAVEL EXNER, Czech Academy of Sciences
Uncommon spectra of periodic quantum graphs: three simple examples

A typical picture of a periodic quantum graph spectrum is that it consists of bands separated by an infinite number of gaps, with the asymptotic band-gap ratio determined by the vertex coupling. We present three simple examples showing that neither of this needs to be true. Specifically, the spectrum may be a Cantor set, number of gaps may be finite, and the band-gap ration may be determined by the topology of the graph. The first example is a chain in a linearly changing magnetic field, the other two use non-magnetic graphs, first a rectangular lattice with a δ coupling, then square and hexagonal lattices with a particular coupling violating the mirror symmetry.

IAN JAUSLIN, IAS Princeton
Liquid Crystals and the Heilmann-Lieb model

In 1979, O.Heilmann and E.H. Lieb introduced an interacting dimer model with the goal of proving the emergence of a nematic liquid crystal phase in it. In such a phase, dimers spontaneously align, but there is no long range translational order. Heilmann and Lieb proved that dimers do, indeed, align, and conjectured that there is no translational order. I will discuss a recent proof of this conjecture. This is joint work with Elliott H. Lieb.

FLORA KOUKIOU, Université de Cergy-Pontoise, France
The entropic signature of freezing in Gaussian mean-field models

The freezing property, initially introduced and studied in the context of random polymers, is recently investigated in a series of works for the log-correlated Gaussian fields and branching Brownian motion. By relating the freezing property with the entropy of the Gibbs measure, we give a new definition of freezing and illustrate this in a general class of Gaussian mean-field models.

MATHIEU LEWIN, CNRS & University of Paris-Dauphine
The Uniform Electron Gas and Density Functional Theory

In this talk I will present recent results obtained with Elliott H. Lieb (Princeton) and Robert Seiringer (Vienna), concerning the Uniform Electron Gas (UEG) and its role in the Local Density Approximation of Density Functional Theory. The UEG is a gas of infinitely many electrons whose density is assumed to be constant everywhere in space. It is defined differently from Jellium, which has a positive constant background but no specific constraint on the density. After constructing the UEG using classical tools from statistical mechanics (thermodynamic limit), I will show how it arises in Density Functional Theory in the limit of a slowly varying density, when minimizing the Levy-Lieb energy functional.

TOMOHIRO SASAMOTO, Tokyo Institute of Technology
Large deviation of a tagged particle in 1D symmetric exclusion process

The one-dimensional symmetric simple exclusion process (SEP) is a simple and well-known stochastic interacting particle system in which many particles perform symmetric random walk with exclusion interaction.

We study the fluctuation properties of a tracer (tagged particle) in the one-dimensional SEP for a uniform density stationary initial condition. The mean position is zero and the anomalous fluctuation of order $t^{1/4}$ has been known for a long time. The large deviation principle (LDP) has been established by Sethuraman and Varadhan in 2013. We will present an exact formula for the rate function for this LDP. Our results can be generalized to the step initial condition with different densities in both direction, and can also be translated to the large deviation of the integrated current at an arbitrary position.

This is a generalization of a previous work on the current at the origin studied by Derrida and Gershenfeld, and also of another work on the single file diffusion of Brownian particles by Krapivsky, Mallick and Sadhu. Our approach uses recently developed techniques to study the one dimensional KPZ equation and asymmetric exclusion process, such as the Bethe ansatz, stochastic duality and nested contour formula for the deformed moments.

Reference: T. Imamura, K. Mallick, T. Sasamoto, Large deviations of a tracer in the symmetric exclusion process, Phys. Rev. Lett. 118, 160601 (2017).

MATĚJ TUŠEK, Czech Technical University in Prague
Location of hot spots in thin curved strips

The celebrated (hot spots) conjecture of Rauch's from 1974 states that any eigenfunction corresponding to the second eigenvalue of the Neumann Laplacian attains its extrema at boundary points only. After reviewing both positive and negative results obtained so far, I show that the maxima and minima of Neumann eigenfunctions of thin tubular neighbourhoods of curves on surfaces are located in terms of the maxima and minima of Neumann eigenfunctions of the underlying curves. In particular, this proves the conjecture for a new class of domains. The talk is based on a joint work with D. Krejčiřík.