### ICMP Contributed Talks 8 Communications libres ICMP 8

## NIELS BENEDIKTER, IST Austria

#### Correlation Energy of the Mean-Field Fermi Gas as an Upper Bound

Interacting Fermi gases are of great technological importance. We consider a model of fermions with bounded compactly supported interaction in the mean-field scaling regime. We incorporate many-body correlations to obtain the first correction to the ground state energy beyond Hartree-Fock theory, as an upper bound. Our approach is to derive an effective quadratic Hamiltonian by bosonizing collective pair excitations. The result is a Gell-Mann–Brueckner–type formula.

# GONTIER DAVID, CEREMADE, Université Paris-Dauphine

Localised Wannier functions in metallic systems

The construction of exponentially localised Wannier functions is an useful theoretical and numerical tool to compute properties of crystals modelled by a periodic Schrödinger operator. In the case where the crystal is an insulator (existence of a spectral gap), this construction is well understood, but the case of metallic systems has been much less explored. In this talk, we show that N energy bands of a metal can be exactly represented by N+1 Wannier functions decaying faster than any polynomial. This is joint work with H. Cornean, D. Monaco and A. Levitt

# JAROSLAV DITTRICH, Nuclear Physics Institute CAS, Rez, Czech Republic

Absolute continuity of the spectra for particles bounded to an infinite planar curve

Non-relativistic quantum particles bounded to an infinite curve in the plane by the attractive contact delta-interaction are considered. The interval between the energy of the transversal bound state and zero is shown to belong to the absolutely continuous spectrum, with possible embedded eigenvalues. The curve is assumed smooth and non-intersecting, asymptotically approaching two different half-lines. The proof is inspired by the proof of Agmon-Kato-Kuroda theorem.

# RAPHAEL DUCATEZ, University Paris Dauphine

Anderson localization for infinitely many interacting particules under Hartree Fock theory

We study the occurrence of Anderson localisation for a system of infinitely many particles interacting with a short range potential, within the ground state Hartree-Fock approximation. We assume that the particles hop on a discrete lattice and that they are submitted to an external periodic potential which creates a gap in the non-interacting one particle Hamiltonian. We also assume that the interaction is weak enough to preserve a gap. We have been able to prove that the mean-field operator has exponentially localised eigenvectors, either on its whole spectrum or at the edges of its bands, depending on the strength of the disorder.

# TRUC FRANÇOISE, Université Grenoble-Alpes, France

The Magnetic Laplacian Acting on Discrete Cusps

We introduce the notion of discrete cusp for a weighted graph. In this context, we prove that the form-domain of the magnetic Laplacian and that of the non-magnetic Laplacian can be different. We establish the emptiness of the essential spectrum and compute the asymptotic of eigenvalues for the magnetic Laplacian. This is a joint work with S. Golenia.

New results about quasi-periodic Schrödinger operators with Liouville frequencies

It has been known since 1970's that one-dimensional discrete quasi-periodic Schroedinger operators with Liouville frequencies don't have eigenvalues. The talk is dedicated to two recent extensions of that result: one of them generalizes it to the multi-dimensional setting (joint work with Arkadi Nemirovski); the other, in a slightly and inevitably weaker form, - to the case, where the sampling function on the torus (which, together with the vector of frequencies and a point of the torus, defines the potential) is only required to be Borel measurable.

#### JORY GRIFFIN, Queen's University

#### Quantum Transport in a Low-Density Periodic Potential: Homogenisation via Homogeneous Flows

The quantum Lorentz gas is a model of conductivity, in which one considers the evolution of some initial wave packet in the presence of a potential consisting of smooth, compactly supported 'scatterers' placed on some infinite, discrete set  $\mathcal{P} \subset \mathbb{R}^d$ . A result of Eng and Erdős tells us that if  $\mathcal{P}$  is suitably random, the solution of the corresponding Shrödinger equation converges in some sense, in the low scatterer density limit, to a solution of the linear Boltzmann equation. This complements the famous work of Boldrighini, Bunimovich and Sinai, who proved an analogous result for the classical Lorentz gas 20 years earlier. We consider the case  $\mathcal{P} = \mathbb{Z}^d$ , and show that up to second-order in the Duhamel expansion the solution agrees with that of the linear Boltzmann equation, yet conjecture that at higher-orders non-Boltzmann contributions arise. Convergence of the second order term forms the bulk of our work, and requires results concerning equidistribution of pieces of horocycles in some homogeneous space. Joint work with Jens Marklof (University of Bristol).

#### JOHN IMBRIE, University of Virginia

#### The Anderson model with discrete disorder

Consider the Schroedinger equation with a random potential taking values 0 or 1. This so-called Anderson-Bernoulli model has been a challenge to mathematicians seeking to understand the localized eigenstates at low energy. Somewhat paradoxically, the technical challenge has been in ruling out eigenfunctions that decay too rapidly. In 2005, Bourgain and Kenig solved this problem in the continuum, by developing quantitative unique continuation estimates. Their ideas don't work on the lattice, however. I will discuss results on the lattice  $Z^d$  for the case of an N-valued potential, with N large, as well as progress on the Bernoulli case, N = 2.

#### KENICHI ITO, University of Tokyo

New methods in spectral theory of N-body Schrödinger operators

We develop a new scheme of proofs for spectral theory of the N-body Schrödinger operators, reproducing and extending a series of sharp results under minimum conditions. The main results include Rellich's theorem, limiting absorption principle bounds, microlocal resolvent bounds, Hölder continuity of the resolvent and a microlocal Sommerfeld uniqueness result. We present a new proof of Rellich's theorem which is unified with exponential decay estimates studied previously only for  $L^2$ -eigenfunctions. Each pair-potential is a sum of a long-range term with first order derivatives, a short-range term without derivatives and a singular term of operator- or form-bounded type. The setup can also include hard-core interactions. Our proofs consist of a systematic use of commutators with 'zeroth order' operator, not like the standard 'first order' conjugate operator in the Mourre theory. In particular, our proofs do not rely on Mourre's differential inequality technique. This talk is based on a recent joint work with T. Adachi, K. Itakura and E. Skibsted.

### **JONAS LAMPART**, CNRS and Université de Bourgogne *Boundary conditions for operators on Fock space*

Hamiltonians for interacting models with variable particle number are often not given by relatively (operator-) bounded perturbations of the noninteracting Hamiltonian, and are thus constructed using quadratic forms or renormalisation. In general, these techniques do not give much information on the domain of self-adjointness. I will present a recent result (arXiv:1803.00872, in collaboration with J. Schmidt) on the definition of such Hamiltonians using generalised boundary conditions, for models in which bosons can be created and annihilated by nonrelativistic "source"-particles. Relevant examples include the Fröhlich model and the Nelson model. The domain of these operators is explicitly given in terms of generalised boundary conditions, similar to those used in the theory of point interactions, that relate different sectors of Fock space. I will introduce the main concepts of this approach and briefly discuss the relation to renormalisation.