Partial Differential Equations
Equations aux dérivées partielles
(Org: **Isabelle Gallagher** (Ecole Normale Supérieure) and/et **Fanghua Lin** (New York University))

SCOTT ARMSTRONG, New York University

JACOB BEDROSSIAN, University of Maryland, College Park
Vortex filaments in the 3D Navier-Stokes equations

We consider the 3D Navier-Stokes equations with initial vorticity given by a measure concentrated on a smooth, non-self-intersecting curve with vorticity of constant magnitude directed along the tangent. Such initial data is an idealization of the coherent structures commonly observed in 3D fluids at high Reynolds numbers known as vortex filaments. The initial data is also exactly in the Koch-Tataru critical regularity class BMO^{-1}, the largest class of initial data for which global well-posedness of mild solutions is known for small data. However, for large data in BMO^{-1}, even local existence of mild solutions is open (note this is in contrast to smaller critical spaces such as L^3). We construct (locally-in-time) mild solutions for vortex filaments of arbitrary circulation which are smooth for all $t > 0$. The solutions we construct are approximately a locally self-similar Gaussian transverse to the filament and are unique and stable in a certain class of such possible solutions. Joint work with Pierre Germain and Benjamin Harrop-Griffiths.

ZAHER HANI, Georgia Institute of Technology
Onset of kinetic behavior for the nonlinear Schrödinger equation

Wave turbulence theory claims that at very long timescales, and in appropriate limiting regimes, the effective behavior of a nonlinear dispersive PDE on a large domain can be described by a kinetic equation called the "wave kinetic equation". This is the wave-analog of Boltzmann's equation for particle collisions. We shall consider the nonlinear Schrodinger equation on a large box with periodic boundary conditions, and explore some of its effective long-time behaviors at time scales that are shorter than the conjectured kinetic time scale, but still long enough to exhibit the onset of the kinetic behavior. (This is joint work with Tristan Buckmaster, Pierre Germain, and Jalal Shatah).

FRÉDÉRIC HÉRAU, Université de Nantes
Short time regularization of diffusive inhomogeneous kinetic equations

In this talk we shall review some results about short time regularization properties of linear or linearized inhomogeneous diffusive kinetic equations. These properties are essentially due to the intrinsec hypoelliptic structure of these equations. This concerns various models, from Kolmogorov to linearized Boltzmann without cutoff equations, and we present some applications in particular for this last model (recent work with I. Tristani and D. Tonon).

JARED SPECK, Massachusetts Institute of Technology
Singularity Formation in General Relativity

The celebrated Hawking–Penrose theorems show that, under appropriate assumptions on the matter model, a large, open set of initial data for Einstein's equations lead to geodesically incomplete solutions. However, these theorems are "soft" in that they do not yield any information about the nature of the incompleteness, leaving open the possibilities that **i)** it is tied to the blowup of some invariant quantity (such as curvature) or **ii)** it is due to a more sinister phenomenon, such as incompleteness due to lack of information for how to uniquely continue the solution (this is roughly known as the formation of a Cauchy horizon). In various joint works with I. Rodnianski, we have obtained the first results in more than one spatial dimension that eliminate the ambiguity for an open set of initial data. In this talk, I will discuss our most recent work, in which we developed a new, more robust analytical framework that allows us to treat initial data exhibiting moderate spatial anisotropy, thus going beyond the regime of nearly spatially isotropic initial data that we treated in earlier works. Our approach applies, for example, to open sets of initial data for the Einstein-vacuum equations in high spatial dimensions and to the Einstein-scalar field system in any number of spatial dimensions. From an analytic perspective, the main theorems are stable blowup results for quasilinear systems of elliptic-hyperbolic PDEs. In this talk, I will provide an overview of these results and explain how they are tied to some of the main themes of investigation by the mathematical general relativity community. I will also discuss the role of geometric and gauge considerations in the proofs, as well as intriguing connections to other problems concerning stable singularity formation.