YRS Contributed Talks 1
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ISMAIL ABOUAMAL, University of Toronto
Fifth-order superintegrable quantum systems separating in Cartesian coordinates.

We consider a two-dimensional quantum Hamiltonian separable in Cartesian coordinates and allowing a fifth-order integral of motion. We impose the superintegrablity condition and find all doubly exotic superintegrable potentials (i.e., potentials V(x, y) = V1(x) + V2(y), where neither V1(x) nor V2(y) satisfy a linear ordinary differential equation), allowing the existence of such an integral. All of these potentials are found to have the Painlevé property. Most of them are expressed in terms of known Painlevé transcendents or elliptic functions but some may represent new higher order Painlevé transcendents.

SÉBASTIEN BERTRAND, Czech Technical University in Prague
Integrable Hamiltonian systems with magnetic field via circular parabolic-type integrals of motion

For integrable Hamiltonian systems, the Hamilton-Jacobi equations can be solved by separation of variables in a suitable coordinate system. In the absence of magnetic field and in low dimension, lists of integrable (and superintegrable) systems together with the coordinate system(s) allowing separation of variables are already well known. However, cases admitting non-zero magnetic fields are less known. Hence, in this presentation, we will consider the integrability and superintegrability of 3D classical Hamiltonian systems. We will focus on natural Hamiltonian systems admitting a static non-zero electromagnetic field mainly via circular parabolic-type integrals of motion. This is a joint work with L. Snobl and A. Marchesiello.

BRIETZKE BIRGER, Institute of Applied Mathematics, University of Heidelberg
The Second Order Correction to the Ground State Energy of the Dilute Bose Gas

We establish the Lee-Huang-Yang formula for the ground state energy of a dilute Bose gas for a broad class of repulsive pair-interactions in 3D as a lower bound. Our result is valid in an appropriate parameter regime of soft potentials and confirms that the Bogolubov approximation captures the right second order correction to the ground state energy. This talk is based on joint work with J. P. Solovej.

LEA BOSSMANN, University of Tübingen
Derivation of the 1d Gross-Pitaevskii equation from the 3d quantum many-body dynamics of strongly confined bosons

We consider the dynamics of N interacting bosons initially exhibiting Bose-Einstein condensation. Due to an external trapping potential, the bosons are strongly confined in two spatial directions, with the transverse extension of the trap being of order ε. The non-negative interaction potential is scaled such that its scattering length is positive and of order $(N/\varepsilon^2)^{-1}$, the range of the interaction scales as $(N/\varepsilon^2)^{-\beta}$ for $\beta \in (0,1]$. We prove that in the simultaneous limit $N \to \infty$ and $\varepsilon \to 0$, the condensation is preserved by the dynamics and the time evolution is asymptotically described by a nonlinear Schrödinger equation in one dimension. The strength of the nonlinearity depends on the interaction and on the shape of the confining potential. For $\beta = 1$, the effective equation is a physically relevant one-dimensional Gross-Pitaevskii equation, where the coupling parameter contains the scattering length of the unscaled interaction. For our analysis, we adapt an approach by Pickl to the problem with dimensional reduction.

Joint work with Stefan Teufel, based on *arXiv:1803.11011* and *arXiv:1803.11026*.

MATTEO CAPOFERRI, University College London
Hyperbolic propagators in curved space

In my talk I will discuss how it is possible, in the spirit of some classical results due to Laptev, Safarov and Vassiliev, to write the propagator of a class of hyperbolic operators on manifolds as one single oscillatory integral with complex-valued phase function, $global$ both in space and in time. In particular, a more refined, geometric version of the method will be presented, in the Riemannian setting: the adoption of a distinguished complex-valued phase function, naturally dictated by the geometric framework, will allow us to visualise the process of circumventing topological obstructions. The microlocal method is explicit and constructive; the calculation of the subprincipal symbol of the propagator enables us to recover asymptotic spectral properties of the operators at hand. I will discuss explicit formulae and recent results for the wave operator.

This is joint work with D. Vassiliev.

THOMAS NORMAN DAM, Aarhus University

Impurities in Bose gasses - recent results on a newly proposed model

In the recent papers ([1]-[4]), the authors investigate a new model describing an impurity in a dilute Bose gas. Compared to the older Frölich model, this new model not only predicts experimental observations better (see [2]), it also has three-particle interactions which are important to model Efimov type physics (see [1],[4]). The aim of this talk is to introduce this new model and present some rigorously proven results which are used in the papers. In particular the authors of the paper [1] used that the global minimum for the mass shell is obtained at total momentum 0 in order to perform variational calculations. We prove this and other results as corollary of a general theorem on positivity improving semigroups which extends results found in the literature.

References

[1] Impurity in a Bose-Einstein Condensate and the Efimov Effect. J. Levinsen, M. M. Parish, and G. M. Bruun. PRL 115, 125302 (2015).

[2] Observation of Attractive and Repulsive Polarons in a Bose-Einstein Condensate. N. B. Jørgensen, L. Wacker, K. T. Skalmstang, M. M. Parish, J. Levinsen, R. S. Christensen, G. M. Bruun, and J. J. Arlt. PRL 117, 055302 (2016).

[3] Strong-coupling Bose polarons in a Bose-Einstein condensate. F. Grusdt, R. Schmidt, Y. E. Shchadilova, and E. Demler. PHYSICAL REVIEW A 96, 013607 (2017).

[4] Quantum dynamics of ultracold Bose polarons. Y. E. Shchadilova, R. Schmidt, F. Grusdt and E. Demler. PRL 117, 113002 (2016).

NICOLO DRAGO, Istituto Nazionale di Fisica Nucleare

Fundamental solutions for the wave operator on static Lorentzian manifolds with timelike boundary

We consider the wave operator on static, Lorentzian manifolds with timelike boundary and we discuss the existence of advanced and retarded fundamental solutions in terms of boundary conditions. By means of spectral calculus we answer this question by studying the self-adjoint extensions of an associated elliptic operator on a Riemannian manifold with boundary (M, g). Assuming that (M, g) is of bounded geometry, this problem can be tackled within the framework of boundary triples. These consists of the assignment of two surjective, trace operators from the domain of the adjoint of the elliptic operator into an auxiliary Hilbert space h. Self adjoint extensions of the underlying elliptic operator are known in one-to-one correspondence with self-adjoint operators Θ on h. For a natural choice of boundary triple, each Θ can be interpreted as the assignment of a boundary condition for the original wave operator. We prove that for each such Θ, it corresponds a unique advanced and retarded fundamental solution. In addition we prove that these share the same structural property of the counterparts associated to the wave operator on a globally hyperbolic spacetime.

LUKAS EMMERT, Ludwig-Maximilian University of Munich
Large Polaron Systems and Bogolubov theory

In this talk we are concerned with the 'neutral' case of the many-polaron system in the Pekar-Tomasevich approximation. In 2015, Benguria, Frank and Lieb showed that in this case the ground state energy goes as $-N^{7/5}$ for large N. They proved an upper bound for the coefficient and conjectured it to be the correct one. In a joint work with Rupert Frank, we establish that this is indeed true by proving the corresponding lower bound. To do so, we study a one-component charged Bose gas with Coulomb interaction and a background with variable charge distribution. Adapting methods of Lieb and Solovej we can justify Bogolubov theory for this model.

ROHAN GHANTA, Georgia Institute of Technology
The Ground-State Wave Function of a Polaron bound to a Coulomb Potential in a Strong Magnetic Field

The Fröhlich polaron is a model of an electron interacting with the quantized optical modes (phonons) of an ionic crystal. We shall consider a (three-dimensional) Fröhlich polaron in a homogeneous magnetic field of strength $B > 0$ and localized in a Coulomb potential. It can be shown using now-standard techniques in quantum field theory that there exists a unique ground-state wave function for all values of B.

It has been argued by E.A. Kochetov, H. Leschke and M.A. Smondyrev (Z. Phys. B. 89, 177-186 (1992)) that as $B \to \infty$ the (three-dimensional) magnetopolaron is "equivalent" to a one-dimensional polaron (without a magnetic field). Kochetov et. al's argument has been placed on a rigorous footing, at least for calculating the ground-state energy, only recently by R.L. Frank and L. Geisinger (Commun. Math. Phys. 338, 1-29 (2015)). Using their technique we see that as $B \to \infty$ the ground-state energy of our model is described by a one-dimensional minimization problem with a delta function potential.

Our contribution is to extend this description also to the wave function: We shall see that as $B \to \infty$ the ground-state wave function–after integrating out its phonon and transverse electron coordinates–converges in a weak sense to the (unique) minimizer of the corresponding one-dimensional problem. Moreover, the one-dimensional minimizer can be evaluated explicitly. Finally, we shall mention some open problems regarding the effective mass and the binding of polarons in strong magnetic fields.

GARRIGUE LOUIS, Université Paris-Dauphine
The Hohenberg-Kohn theorem

Density functional theory is a very active field of research, and is of great importance in today's science. It is founded by the Hohenberg-Kohn theorem, a strinking result stating that all the information of a quantum mechanical system is contained in its ground-state one-body density only. We provide a first rigorous proof of this theorem in a physically relevant setting. It turns out that this reduces to prove a strong unique continuation property for many-body Schrodinger operators that we showed using recent Carleman estimates.

ANGELO LUCIA, QMATH, University of Copenhagen
A limitation on the asymptotic decay of vanishing spectral gaps

The spectral gap of quantum many-body systems is one of the most fundamental quantities determining much of the properties of the low-energy physics, and understanding its behavior is a particularly hard problem. Even in the case of a gapless system, where the spectral gap goes to zero as the system size increases, it is important to understand the rate at which it vanishes. For the case of finite-range, frustration free Hamiltonians on a spin lattice of arbitrary dimension, we show that in the gapless case, the spectral gap on regions of diameter n is at most $o\left(\frac{\log(n)^{2+\varepsilon}}{n}\right)$ for any positive ε. Any slower decay rate is ruled out, as it would imply a constant spectral gap, via a recursive strategy.

CHRISTOPHER MAHADEO, University of Saskatchewan
Wrapping in two bands

Topology has proven to be a crucial tool for studying conductance in materials. Topology manifests in the following way: metals and insulators are classified by the existence of an energy gap, as the latter possess a gap between valence and conduction bands while the former do not. The eigenstates and Hamiltonian of a crystal are indexed by momenta and thus give rise to a vector bundle structure over the Brillouin zone. Electronic properties of an insulator are then governed by the topological properties of the subbundle corresponding to valence bands. These bands can possess non-trivial topologies that can be realized as an obstruction to define wave functions over the entire Brillouin zone using a single phase convention. We show that in a two-band insulator this non-trivial topology manifests as a winding number. We explore the potential application of "abelianization" to the classification of topological materials.

YULIA MESHKOVA, Chebyshev Laboratory, St. Petersburg State University
Operator error estimates for homogenization of elliptic and parabolic systems

We consider a matrix strongly elliptic second order differential operator acting in a bounded domain with the Dirichlet boundary condition. The operator is self-adjoint. Coefficients are periodic and oscillate rapidly. We study the behavior of solutions of the corresponding elliptic and parabolic systems in the small period limit. The results can be written as approximations of the resolvent and the semigroup in $L_2 \to L_2$ and $L_2 \to H^1$ operator norms. So, the estimates of this type are called operator error estimates in homogenization theory. The talk is based on a joint work with T. A. Suslina.

ATMN PATEL, University of Waterloo
An Analytic Approach to Quantum Shannon Theory

We characterize the conditions under which the quantum channel capacities are analytic. If the optimization over input states is restricted to positive operators, the quantum and classical channel capacities are analytic on the set of channels. This proof arises naturally from Danskin's Theorem which characterizes the Gatiéaux differential of an optimization function and a number of standard theorems from complex function theory. We calculate the general power series expansion of the n-shot classical and quantum capacity using standard techniques from functional calculus. This can be used to directly compute the n-shot quantum capacity without any optimization given that the center of the expansion is a degradable channel. As a direct application of computing these derivatives, we develop a necessary and sufficient criterion for n-shot superactivation. It is also shown that the power series expansion can be inverted using Lagrange's Inversion Theorem to recover the optimal input state.

RENAUD RAQUÉPAS, McGill University
Control theory in the study of mixing properties of networks of oscillators interacting with thermal baths

We discuss the use of elements of control theory as an alternate tool for showing exponential convergence to a unique stationary measure for certain classes of networks of classical oscillators interacting with thermal baths at different temperatures. With the system of oscillators expressed in the form $\mathrm{d}X_t = AX_t\,\mathrm{d}t + F(X_t)\,\mathrm{d}t + B\,\mathrm{d}W_t$ in $\mathbf{R}^d$, where A encodes the harmonic part of the force and $-F$ corresponds to the gradient of the anharmonic part of the potential, the hypotheses under which we obtain exponential mixing are the following: A is dissipative, the pair (A, B) satisfies the Kalman condition, F grows sufficiently slowly at infinity (depending on the dimension d), and the vector fields in the equation of motion satisfy a weak Hörmander condition in at least one point of the phase space.

JESSICA SANTIAGO, Victoria University of Wellington
Temperature distribution in a rotating universe

In this talk we will review Tolman's relation for temperature gradients in thermal equilibrium states. We will do that by presenting a simplified derivation of this effect, based on the relativistic Euler equation, which will lead naturally to an extension of Tolman-like thermal gradients to the case of stationary spacetimes. We will then explore the thermodynamics of a rotating universe: asking what local thermometers in a rotating disk will measure and what co-moving observers will see. We aim to show how gravity's universality, gravitational redshifts and the observer-dependency of temperature are connected.

CLÉMENT TAUBER, ETH Zürich
Bulk-edge correspondence for Floquet topological insulators

Floquet topological insulators describe independent electrons on a lattice driven out of equilibrium by a time-periodic Hamiltonian, beyond the usual adiabatic approximation. In dimension two such systems are characterized by integer-valued topological indices associated to the unitary propagator, alternatively in the bulk or at the edge of a sample. In this talk I will give new definitions of the two indices, relying neither on translation invariance nor on averaging, and show that they are equal. In particular disorder and defects are intrinsically taken into account. This is a joint work with Gian Michele Graf.

ANH-KHOI TRINH, McGill University
$\mathcal{N}=4$ *Supersymmetric Yang-Mills Correlators from Supergravity*

Computational techniques to study $AdS_5 \times S^5$ supergravity are limited by standard quantum field theory techniques in curved backgrounds. One may hope to bootstrap non-trivial results from the $\mathcal{N}=4$ supersymmetric Yang-Mills (SYM) conformal field theory (CFT) dual since conformal theory correlators are fully specified by their spectrum and operator product expansion coefficients. Thanks to the recent discovery of inversion integrals based on the classic Froissart-Gribov dispersion inversion formula, one can now extract the CFT data as an analytic function of spin. By utilizing general bootstrap techniques, we present a framework to analytically study the large-N supergravity limit of $\mathcal{N}=4$ SYM by describing the spectrum of intermediate double-trace operator exchange from a four-point function half-BPS scattering process.

PETRI TUISKU, University of Helsinki
2D FK-Ising interface scaling limit in annular domain

We present an on-going joint work with Antti Kemppainen (University of Helsinki) to study the scaling limit of the boundary interface of the two dimensional square lattice Fortuin-Kasteleyn (FK, also known as random cluster model) Ising model (that is, cluster weight $q=2$ case) in criticality (that is, edge weight $p=p_c=\frac{\sqrt{2}}{\sqrt{2}+1}$ case) in annular planar domains. Our goal is to prove that when appropriate boundary conditions are assigned to the model, the resulting boundary interface converges in the scaling limit to Schramm's chordal SLE$(16/3)$. This result is known (Chelkak, Duminil-Copin, Hongler, Kemppainen, Smirnov) for planar simply connected domains and our work generalizes this result.

JOONAS TURUNEN, University of Helsinki
Critical Ising model on infinite random triangulation of the half-plane

We consider the Boltzmann random triangulation of a polygon coupled to Ising model with Dobrushin boundary conditions, which can be viewed as a discrete model of Liouville quantum gravity with matter. We derive an explicit expression of the partition function of the finite triangulations at the critical point. Especially we find an asymptotic perimeter exponent different from the one of the pure gravity universality class. Moreover, we show that the local limit of the Boltzmann Ising-triangulations exists in the sense of Benjamini-Schramm as the boundary size tends to infinity one segment after another, which by using a peeling process can be characterized as an explicit Ising-decorated infinite random triangulation. We also show that the infinite component of the interface only touches the boundary a finite number of times as the boundary size tends to infinity. Scaling limits of the perimeter fluctuations associated with the peeling process as well as the size of the interface are also obtained. This is a joint work with Linxiao Chen (University of Helsinki).