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H. M. BHARATH, Georgia Institute of Technology

Non-Abelian Geometric Phases Carried by the Spin Fluctuation Tensor

The geometric information of the trajectory along which a physical system is transported is often accumulated in the system's gauge variables, and is known as geometric phase. This has been a subject of intense study, both theoretically and experimentally over the past three decades. Here, we develop a new non-Abelian geometric phase that is accumulated in the second order spin moments of a quantum spin system.

The expectation values of the first and second moments of the quantum mechanical spin operator can be used to define a spin vector and spin fluctuation tensor respectively. The former is a vector inside the unit ball in three space, while the latter is represented by an ellipsoid. By considering transport of the spin vector along loops in the unit ball we show that the spin fluctuation tensor picks up geometric phase information [1]. For the physically important case of spin one, the geometric phase is formulated in terms of an SO(3) operator. Loops defined in the unit ball fall into two classes: those which do not pass through the origin and those which pass through the origin. The former class of loops subtend a well defined solid angle at the origin while the latter do not and the corresponding geometric phase is non-Abelian. To deal with both classes, we introduce a *generalized solid angle*, which helps to clarify the interpretation of the geometric phase information.

[1]. H. M. Bharath, "Non-Abelian geometric phases carried by the spin fluctuation tensor", arXiv: 1702.08564

SOROUR KARIMI DEHBOKRI, Technische Universitat Braunschweig

Renormalization Group flow

Almost two decades ago, Renormalization Group flow defined by the smooth Feshbach-Schur map was shown by V. Bach, Chen, Fröhlich, and Sigal to possess a codimension-one contractivity property. This contractivity insures that the iterative application of R_ρ (the Renormalization Transformation depends on a scaling parameter ρ) generates a (time-discrete) dynamical system on D (small ball of Banach space of operators, that is the domain of definition of the RG map) with a fixed point manifold of dimension one. Now we improved scheme that is (fully) contracting and has no marginal directions anymore. This allows for characterizing the properties on the fixed point much more precisely. This is joint work with V. Bach

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PAWEŁ DUCH, Jagiellonian University

Adiabatic limit and vacuum state in Epstein-Glaser approach to perturbative quantum field theory

The fundamental objects of Epstein-Glaser approach to perturbative quantum field theory are the time-ordered products of polynomials in the basic fields and their derivatives. Their construction is carried out in the position space and does not require the introduction of any ultraviolet regularization. Using the time-ordered products one can easily define the scattering matrix, the interacting fields and other objects of interests in the interacting theory in which the coupling constant is replaced by a Schwartz function called the switching function. The switching function plays the role of the infrared regulator, which is removed by taking the adiabatic limit.

In the talk, I will outline my recent results about the existence of the so-called weak adiabatic limit. The result allows to construct the Wightman and Green functions in a large class of models, which includes all models with interaction vertices of dimension 4. The existence of the weak adiabatic limit can be also used to define a vacuum state (a real, normalized, positive, Poincaré-invariant functional) on the algebra of interacting fields constructed by means of the algebraic adiabatic limit.

FEDERICO FALDINO, University of Genova

On interacting KMS states in pAQFT: Stability, Relative Entropy and Entropy Production

In this talk, we analyze the stability and return to equilibrium properties of the interacting KMS states built by Fredenhagen and Lindner for a scalar field theory in the framework of parturbative Algebraic Quantum Field Theory [1]. In particular, we show that this properties hold for compactly supported potentials, while they fail if the adiabatic limit is considered. This failure led to the definition of a Non-Equilibrium Steady State in pAQFT [2].

Furthermore, in order to study this new non-equilibrium state, we define relative entropy and of entropy production in the framework of pAQFT [3].

Bibliography:

[1] K. Fredenhagen, F. Lindner - "Construction of KMS States in Perturbative QFT and Renormalized Hamiltonian Dynamics". Commun. Math. Phys. 332 – 895 (2014). [2] N. Drago, F. Faldino, N. Pinamonti - "On the stability of KMS states in perturbative algebraic quantum field theory". Commun. Math. Phys 357, Issue 1 (2018) 267-293. [3] N. Drago, F. Faldino, N. Pinamonti - "Relative Entropy and Entropy Production for Equilibrium States in pAQFT". ArXiv:[1710.09747] (2017).

XIAO HE, Département de mathématiques et de statistique, Université Laval
How to add "ghosts" in BRST reduction ? —-A remark on semi-infinite cohomology

The rigorous mathematical definition of semi-infinite cohomology was introduced by B. Feigin in 1984, it can be considered as the counterpart of BRST reduction in physics. Unlike ordinary Lie algebra cohomology, computing semi-infinite cohomology requires that the Lie algebra admits a semi-infinite structure. Roughly speaking, a semi-infinite structure is a Lie algebra module structure on the space of semi-infinite forms, and the requirement of such a structure is to make the BRST differential nilpotent, i.e., square zero, which is essential in cohomology theory.

What about if the Lie algebra admits no semi-infinite structure? One way to adjust this is to consider some one-dimensional central extension, which is called cancellation of anomalies in physics. Another way is, as the physicists already did, to add more "ghosts", hence to modify the BRST complex, and then to make a deformation of the BRST differential to make it nilpotent.

In my talk, I will take affine W-algebras as the example, to explain how to add "ghosts" and how to modify the BRST differential in a rigorous mathematical way. As a byproduct, we will give a uniformed definition of affine W-algebras in general nilpotent element case. This is based on our recent work "*A remark on semi-infinite cohomology.*" arXiv:1712.05484.

ATSUHIDE ISHIDA, Tokyo University of Science
Propagation property and inverse scattering for fractional powers of the negative Laplacian

We define the fractional power of the negative Laplacian as the self-adjoint operator acting on $L^2(\mathbb{R}^n)$:

$$H_{0,\rho} = (-\Delta)^\rho/(2\rho)$$

for $1/2 \leqslant \rho \leqslant 1$ where $\Delta = \sum_{j=1}^n \partial_{x_j}^2$. If $\rho = 1$, $H_{0,1}$ denotes the free Schrödinger operator $H_{0,1} = -\Delta/2$. On the other hand, if $\rho = 1/2$, then $H_{0,1/2}$ denotes the massless relativistic Schrödinger operator $H_{0,1/2} = \sqrt{-\Delta}$. We study one of the propagation estimates (Enss-type estimate) for the free dynamics $e^{-itH_{0,\rho}}$ and try to apply this estimate to inverse scattering for $\rho > 1/2$ by using the Enss-Weder time-dependent method. We report that the high velocity limit of the scattering operator uniquely determines the short-range interactions. This work was partially supported by the Grant-in-Aid for Young Scientists (B) No.16K17633 from JSPS.

MICHAL JEX, Karlsruher Institut für Technologie
Revisiting Lieb-Thirring Inequalities

The moment inequalities due to Lieb and Thirring are effective tools in the operator theory. Especially the one for the sum of the (negative) eigenvalues of a Schrödinger operator, since, by duality, it is equivalent to a lower bound for the kinetic energy of Fermions, which is exactly of the right semi-classical Thomas–Fermi type.

Based on ideas of Rumin, we show a novel approach of proving the Lieb-Thirring inequalities for the operator $H = |p|^k - U$ with arbitrary $k > 0$ in any dimension d. The obtained constants are improvements of currently known results in all cases, in particular, for $k = 2$.

The other advantage is that the derived factors relating our inequality to semiclassical ones, that is, the quotient of our constants divided by the semi-classical guess, are uniformly bounded for all k and d by e.

We also estimate number of negative eigenvalues for the operator H with dimension $d > k$. Factoring out the semiclassical estimate on the number of bound states yields a uniformly bounded estimate converging to e^2 for large dimensions. These results work for all k and do not use an extension of the bounds to operator-valued potentials and the induction in the dimension trick of Laptev and Weidl, which works only for $k = 2$.

This seems to be the first time that one can prove universal bounds without using some type of induction in the dimension argument. However, for $k = 2$ one can do this and we get bounds which improving the bounds for small values of d in this case.

HANS KONRAD KNÖRR, Aalborg Universitet

On the adiabatic behaviour of a bound state when diving into the continuous spectrum

The survival probability of a bound state is studied when an external potential varies smoothly and adiabatically in time. The initial state corresponds to a discrete eigenvalue which dives into the continuous spectrum and re-emerges from it as the potential is varied in time and finally returns to its initial value. The main result is that the survival probability of this bound state vanishes in the adiabatic limit. The methods used in the proof are quite robust and may be adopted to cover a large class of operators, including Schrödinger and Dirac operators. This talk is based on joint work with H. Cornean, A. Jensen and Gh. Nenciu.

MARKUS LANGE, Karlsruhe Institute of Technology

On Asymptotic Expansions for Spin Boson Models

We consider expansions of eigenvalues and eigenvectors for a class of models known as generalized spin boson models. We prove existence of asymptotic expansions for the ground state and the ground state energy to arbitrary order. We need a mild but very natural infrared assumption, which is weaker than the assumption usually needed for other methods such as operator-theoretic renormalization to be applicable. The result complements known analyticity properties.

NIKOLAI LEOPOLD, Institute of Science and Technology Austria

Mean-field Dynamics for the Nelson model with Fermions

The Nelson model (with ultraviolet cutoff) describes a quantum system of non-relativistic identical particles coupled to a quantized scalar field. In this talk, I would like to discuss its time evolution in a mean-field limit of many fermions which is coupled to a semiclassical limit. At time zero, we assume that the bosons of the radiation field are in a coherent state and that the state of the fermions is given by a Slater determinant, whose reduced one-particle density matrix is an orthogonal projection with semiclassical structure. At later times and in the limit of many fermions it can be proven that the fermion state remains close to a slater determinant and that the time evolution is approximately described by the fermionic Schrödinger-Klein-Gordon equations. I will introduce the mentioned models and explain our main theorem. The talk is based on work in progress with Sören Petrat.

TIANSHU LIU, The University of Melbourne

osp(1—2) Minimal Models And Their Coset Construction

Conformal field theory is an essential tool of modern mathematical physics with applications to string theory and to the critical behaviour of statistical lattice models. The symmetries of a conformal field theory include all angle-preserving transformations. In two dimensions, these transformations generate the Virasoro algebra, a powerful symmetry that allows one to calculate observable quantities analytically. The construction of one family of conformal field theories from the affine Kac-Moody algebra sl(2) were proposed by Kent in 1986 as a means of generalising the coset construction to non-unitary Virasoro minimal models, these are known as the Wess-Zumino-Witten models at admissible levels. This talk aims to illustrate, with the example of the affine Kac-Moody superalgebra osp(1—2) at admissible levels, how the representation theory of a vertex operator superalgebra can be studied through a coset construction. The method allows us to determine key aspects of the theory, including its module characters, modular transformations and fusion rules.

SIMON MAYER, Institute of Science and Technology Austria
The free energy of a dilute 2d Bose gas

We consider a two-dimensional interacting Bose gas in a homogeneous setting. The two-body interaction potential is assumed to be non-negative and of finite scattering length a. Under these quite general assumptions, we are able to obtain an asymptotic expansion formula of the free energy of the system at non-negative temperature in the dilute limit $a^2\rho \ll 1$, where ρ is the density. In the limit of zero temperature, the formula reduces to the asymptotic ground state energy which is an earlier result by Lieb and Yngvason (2001). Our work extends the corresponding result in three dimensions proved by R. Seiringer (2008) and J. Yin (2010).

KONSTANTIN MERZ, Ludwig-Maximilians-Universität München
On the Strong Scott Conjecture in the Chandraskehar Model

We consider large neutral atoms of atomic number Z. For such atoms the speed of electrons close to the nucleus is a substantial fraction of the speed of light c. Thus, a relativistic description is necessary. We model the atom by the pseudo-relativistic Hamiltonian of Chandrasekhar.

Our main result is the convergence of the suitably rescaled one-particle ground state density in each angular momentum channel: it converges on distances $1/Z$ from the nucleus to the corresponding density of the one-particle hydrogenic Chandrasekhar operator. This proves a generalization of the strong Scott conjecture for relativistic atoms.

The proof uses the Scott correction, i.e., the two term expansion of the ground state energy (Solovej, Sørensen, and Spitzer and Frank, Siedentop, and Warzel), and a new equivalence of Sobolev norms generated by the free and the hydrogenic Chandrasekhar operators.

The result underscores that relativistic effects occur close to the nucleus and that self-interactions of the innermost electrons are negligible.

LUCA NENNA, (CEREMADE) Université Paris-Dauphine
Multi-Marginal Optimal Transport in Quantum Mechanics

The strong-interaction limit of the Hohenberg-Kohn functional defines a multi-marginal optimal transport problem with Coulomb cost. From physical arguments, the solution of this limit is expected to yield strictly-correlated particle positions, related to each other by co-motion functions (or optimal maps), but the existence of such a deterministic solution in the general three-dimensional case is still an open question. A conjecture for the co-motion functions for radially symmetric densities was presented in Phys. Rev. A 75, 042511 (2007), and later used to build approximate exchange-correlation functionals for electrons confined in low-density quantum dots. In this talk I will revisit the whole issue both from the formal and numerical point of view (by means of the entropic regularisation of Optimal Transport), finding that even if the conjectured maps are not always optimal, they still yield an interaction energy (cost) that is numerically very close to the true minimum.

ALESSANDRO OLGIATI, SISSA, Trieste
Ground state properties of mixtures of condensates

I will present a rigorous proof of the ground state energy asymptotics for multi-component condensates. Such systems consist of multiple species of identical bosons, and their mathematical study has become topical very recently.

I will show that, both in the mean field and Gross-Pitaevskii regime, the leading order of the ground state energy is captured by the minimum of a suitable one-body non-linear functional. Moreover, the ground state exhibits condensation in the sense of reduced density matrices.

In the mean field regime, by an implementation of Bogoliubov theory, we are also able to compute the next-to-leading order of the ground state energy asymptotics, and to prove a norm approximation for the ground state.

All our results hold under a miscibility condition, as is often called in physics literature, that allows us to prove uniqueness of the minimizer of the non-linear theory.

This is a joint work with Alessandro Michelangeli and Phan Thành Nam.

ROBERT RAUCH, Technische Universität Braunschweig
Orthogonalization of Fermion k-Body Operators and Representabilty

The reduced k-particle density matrix (k-RDM) of a density matrix ρ on fermion Fock space $\mathcal{F}$ can be defined as the image under the orthogonal projection

$$\pi_k : \mathcal{L}^2(\mathcal{F}) \to \mathcal{O}_k \subset \mathcal{L}^2(\mathcal{F})$$

onto the space $\mathcal{O}_k$ of k-body observables on $\mathcal{F}$ within the space of Hilbert-Schmidt operators $\mathcal{L}^2(\mathcal{F})$. A proper understanding of π_k is intimately related to the *representability problem*, a long-standing open problem in computational quantum chemistry, which amounts to give a computationally efficient characterization of the cone $\pi_k(\mathcal{P})$ of *representable* k-RDMs, where $\mathcal{P}$ denotes the cone of positive trace-class operators on $\mathcal{F}$.

The goal of this joint work with V. Bach is the derivation of new representability conditions and the characterization of π_k in the finite-dimensional case. We have recently completed the first step towards this goal by explicitly constructing a distinguished orthonormal basis of $\mathcal{L}^2(\mathcal{F})$ which restricts to a basis adapted to the flag $0 \subsetneq \mathcal{O}_1 \subsetneq \mathcal{O}_2 \subsetneq \cdots$ of k-body observables. This orthonormal basis serves as a tool for the study of the cone $\pi_k(\mathcal{P})$ of representable density-matrices.

LUKAS SCHIMMER, University of Copenhagen
Distinguished self-adjoint extensions of operators with gaps

Semibounded symmetric operators have a distinguished self-adjoint extension, the Friedrichs extension. The eigenvalues of the Friedrichs extension are given by a variational principle that involves only the domain of the symmetric operator. Although Dirac operators describing relativistic particles are not semibounded, the Dirac operator with Coulomb potential is known to have a distinguished extension. In this talk I will relate this extension to a generalisation of the Friedrichs extension to the setting of operators satisfying a gap condition. In addition I will prove, in the general setting, that the eigenvalues of this extension are also given by a variational principle that involves only the domain of the symmetric operator.

This is joint work with Jan Philip Solovej and Sabiha Tokus.

JACOB SHAPIRO, ETH Zurich
The topology of non-interacting electrons in strongly-disordered chiral chains

We explore the strongly-disordered regime of chiral one dimensional systems which may exhibit topological properties. We extend the usual definitions of the topological invariants given in the spectral gap regime to the mobility gap regime, show the connection to localization, and prove the bulk-edge duality in both spectral and mobility gap regimes.

(Based on joint work with G. M. Graf)

YUKIHIDE TADANO, The University of Tokyo
Long-range scattering theory for discrete Schrödinger operators

In this talk, we consider discrete Schrödinger operators $H = H_0 + V$ on periodic lattices including the square lattice $\mathbb{Z}^d$ and the hexagonal lattice. We prove that we can construct a long-range scattering theory for a pair of H_0 and H if the perturbation V is a long-range potential. More precisely, we construct time-independent (or Isozaki-Kitada) modifiers $W^{\pm}(\Gamma) = \text{s-lim}_{t\to\pm\infty} e^{itH} J e^{-itH_0} E_{H_0}(\Gamma)$, where Γ is any open set of $\sigma(H_0)$ away from the threshold energies, and prove that they are asymptotically complete. The above modifiers are constructed from a solution of the corresponding eikonal equation on the outgoing and incoming regions of $T^*\mathbb{T}^d$. The proof is analogous to that in the paper by Isozaki and Kitada in 1985; we use the stationary phase method and the Enss method for the proof of the existence and the completeness of $W^{\pm}(\Gamma)$, respectively. The proof for the hexagonal lattice is more complicated, because we need the diagonalization of H_0 and additional argument due to the corresponding Hilbert space $\ell^2(\mathbb{Z}^2;\mathbb{C}^2)$.

TOSHIMITSU TAKAESU, Gunma University
Ground States of Quantum Electrodynamics with Cutoffs

We investigate a system of a Dirac field coupled to a quantized radiation field in the Coulomb gauge. The Hilbert space for the system is defined by $\mathcal{F}_{\rm QED} = \mathcal{F}_{\rm Dirac} \otimes \mathcal{F}_{\rm rad}$ where $\mathcal{F}_{\rm Dirac}$ is the fermion Fock space over $L^2(\mathbb{R}^3_{\mathbf{p}};\mathbb{C}^4)$ and $\mathcal{F}_{\rm rad}$ is the boson Fock space over $L^2(\mathbb{R}^3_{\mathbf{k}} \times \{1,2\})$. The total Hamiltonian is defined by

$$H_{\rm QED} = H_{\rm Dirac} \otimes I + I \otimes H_{\rm rad} + \kappa_{\rm I} \sum_{j=1}^{3} \int_{\mathbb{R}^3} \chi_{\rm I}(\mathbf{x}) \left(\psi^*(\mathbf{x}) \alpha^j \psi(\mathbf{x}) \otimes A^j(\mathbf{x}) \right) d\mathbf{x} + \kappa_{\rm II} \int_{\mathbb{R}^6} \frac{\chi_{\rm II}(\mathbf{x})\chi_{\rm II}(\mathbf{y})}{|\mathbf{x}-\mathbf{y}|} \left(\psi^*(\mathbf{x})\psi(\mathbf{x})\psi^*(\mathbf{y})\psi(\mathbf{y}) \otimes I \right) d\mathbf{x} d\mathbf{y},$$

on the Hilbert space. Here $H_{\rm Dirac}$ and $H_{\rm rad}$ denote the field energy Hamiltonians, $\psi(\mathbf{x}) = (\psi^l(\mathbf{x}))_{l=1}^4$ and $\mathbf{A}(\mathbf{x}) = (A^j(\mathbf{x}))_{j=1}^3$ denote the field operators with ultraviolet cut-offs, and $(\alpha^j)_{j=1}^3$ denote the Dirac matrices. The total Hamiltonian is self-adjoint and bounded from below. We assume spatially localized conditions and momentum regularity conditions. Then it is proven that the total Hamiltonian has a ground state for all values of coupling constants. In particular, its multiplicity is finite.

ARNAUD TRIAY, Ceremade - Université Paris-Dauphine
Derivation of the dipolar Gross-Pitaevskii energy

The Gross-Pitaevskii theory effectively describes the ground state and the evolution of a dilute and ultracold gas of bosons. A very vast literature exists on the derivation of this theory from the principles of quantum mechanics, nevertheless it remains a challenging task is to address the case of non-positive interactions such as dipole-dipole potentials. We will present how, by using the so-called quantum de Finetti theorem, we can we show the convergence of the ground state and of the ground state energy of the (linear) N body Hamiltonian towards those of the dipolar GP functional. The latter, in addition to the usual cubic nonlinearity, has a long range dipolar term $K \star |u|^2 |u|^2$. Our results hold under the assumption that the two-particle interaction is scaled in the form $N^{3\beta-1} w(N^\beta x)$ for some $0 \le \beta < \beta_{max}$ with $\beta_{max} = 1/3 + s/(45+42s)$ where s is related to the growth of the trapping potential. arXiv:1703.03746