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Communications libres YRS 3
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ANDREA AGAZZI, Duke University
Large deviations theory for chemical reaction networks

At the microscopic level, the dynamics of arbitrary networks of chemical reactions can be modeled as jump Markov processes whose sample paths converge, in the limit of large number of molecules, to the solutions of a set of algebraic ordinary differential equations. Fluctuations around these asymptotic trajectories and the corresponding phase transitions can in principle be studied through large deviations theory in path space, also called Wentzell-Freidlin (W-F) theory. However, the specific form of the jump rates for this family of processes does not satisfy the standard regularity assumptions imposed by such theory, and weaker conditions need to be developed to deal with the framework at hand.

Using tools of Lyapunov stability theory we design sufficient conditions for the applicability of large deviations estimates to the asymptotics of the Markov process at hand. We then translate such conditions in terms of the topological structure of the chemical reaction network. This ultimately allows to define a large class of chemical reaction systems to which the estimates of interest can automatically be applied.

SIMON BECKER, University of Cambridge
Graphene in magnetic fields

Using semiclassical (with the strength of the magnetic field as the small parameter) and spectral methods we study the properties of graphene in magnetic fields with a simple quantum graph model. From the semiclassical analysis, we obtain a geometric description of the density of states which can be used to study magnetic oscillations such as the de Haas-van Alphen effect. We finally provide an outlook on 2D models for graphene in a magnetic field and related configurations such as bilayer graphene. This talk is based on joint work with M Zworski, as well as R Han and S Jitomirskaya.

MICHAEL ANTHONY BISHOP, California State University, Fresno
Spectral gaps for the Two-Species Product Vacua and Boundary States models on the d-dimensional lattice

We study the two-species Product Vacua and Boundary States (PVBS) models on the integer lattice $\mathbb{Z}^d$ and prove the existence and non-existence of a spectral gap for all choices of parameters. The PVBS models are spin-1 quantum spin systems which are translation-invariant, frustration-free, and composed of nearest-neighbor non-commuting interactions with both an exclusion property and an interchange interaction between particle species. These models serve as possible representatives of families of automorphically equivalent gapped quantum spin-1 systems on $\mathbb{Z}^d$.The main result is that the two-species PVBS Hamiltonians have a positive spectral gap when gapped on both of the single-species subspaces and are gapless if gapless on either single-species subspace. The addition of a new particle species does not create any new gapless phases.

JACKY CHONG, University of Maryland, College Park
Dynamics of Large Boson Systems with Attractive Interaction and a Derivation of the Cubic Focusing NLS in $\mathbb{R}^3$.

We consider a N-particle boson system with two-body interaction $N^{3\beta-1}v(N^\beta x)$ where $v \in C_0^\infty$ for some range of β. We extend the results of Grillakis et al. in *Comm. Math. Phys.,* (2013) and Kuz in *Differential Integral Equations*, (2017) regarding second-order correction to mean-field evolution of systems with repulsive interaction to the case of attractive interaction for $0 < \beta < \frac{1}{2}$. The two key ingredients used to extend to this case of attractive interaction are the proofs of the uniform global wellposedness of solutions to a family of Hartree-type equations and the corresponding L^∞-decay estimates on the solutions. Inspired by the recent works Pickl *J. Stat. Phys.* (2010), Chen et al. in *Theo. and Math. Phys.,* (2013) and Chen et al. in

Arch. Ration. Mech. Anal. (2016), we also provide both a derivation of the focusing cubic nonlinear Schrodinger equation (NLS) in 3D from the many-body Boson system and its rate of convergence toward mean field.

MAXIMILIAN DUELL, Technische Universität München
Asymptotic Completeness and N-Particle Scattering in Wedge-local QFT

I will outline my recent construction of scattering states with arbitrarily large number of particles in the general operator-algebraic setting of massive wedge-local quantum field theory. In this context wedge-geometrical obstructions invalidate conventional Haag-Ruelle arguments beyond the two-particle case. It will be explained how such limitations are overcome in my work via wedge-duality arguments. Recently constructed wedge-local models exhibiting non-trivial scattering on four-dimensional Minkowski space-time (Grosse-Lechner, Buchholz-Lechner-Summers) can be investigated with my construction regarding particle-phenomenological features such as asymptotic completeness. (partially based on arxiv:1711.02569, accepted in CMP)

FIONA GOTTSCHALK, Technische Universität Braunschweig
Infrared Bounds for Ferromagnetic Models without Translation Invariance

Subject of our considerations are the infrared bounds established by Fröhlich, Simon, and Spencer in 1976 for classical statistical mechanical lattice systems with a continuous symmetry. These bounds imply the existence of a phase transition at sufficiently low temperature in three or more spatial dimensions.
Instead of using reflection positivity, we reproduce and generalize the result by means of the Helffer-Sjöstrand Formula and the Witten Laplacian, allowing for weaker assumptions on translation invariance. This is joint work with V. Bach.

JOSEPH GRENIER, Louisiana State University
Quantum Field Theory on the Sphere

In 2016, Dan Freed and Michael Hopkins explored the concept on Reflection Positivity in Topological Quantum Field Theory. In this talk, a TQFT will be developed on the sphere (S^2) through the use of Reflection Positivity and be contrasted with a constructive QFT on S^2.

ALEXANDER HACH, TU Braunschweig
Suppression of decoherence of a Spin-Boson system by time-periodic control

We consider a finite-dimensional quantum system coupled to a bosonic radiation field and subject to a time-periodic control operator. Assuming the validity of a certain dynamic decoupling condition we study the system's time evolution with regard to the non-interacting dynamics. For sufficiently small coupling constants g and control periods T we show that a certain deviation between coupled and uncoupled propagator as a measure for quantum decoherence may be estimated by $\mathcal{O}(gt\,T)$. Our approach relies on the notion of Kato stability and general theory of non-autonomous linear evolution equations.

HIDEKI INOUE, Nagoya University
Topological Levinson's theorem for inverse square potentials with infinitely many bound states

Levinson's theorem is a surprising result in quantum scattering theory, which is originally established by N. Levinson for a spherically symmetric potential in 1949. It is a relation between an expression coming from the scattering part of a quantum system and the number of bound states of that system. In 2007, J. Kellendonk and S. Richard gave a topological interpretation of this relation as an index theorem by using K-theory for C*-algebras. More precisely, if the number of bound states are finite, then the Møller wave operators are Fredholm and an index theorem is proved for them via a Toeplitz extension of $C(\mathbb{T})$ by the set of compact operators.

The common feature of these investigations is that the number of bound states of the underlying system is finite, but what happens if infinitely many bound states are involved? In this talk, we consider a model consisting in Schrödinger operators $H_{m,\kappa}$ on a half-line with a potential of the form r^{-2}. Depending on the parameters (m,κ), the wave operators are either Fredholm, semi-Fredholm or almost-periodic operators. We introduce new algebraic frameworks and establish meaningful relations for each cases, which correspond to the topological Levinson's theorem for this model. The resulting relations are concrete realizations of several famous index theorems, e.g. Atiyah's L^2-index theorem.

This talk is based on a joint work with S. Richard.

JINYEOP LEE, KAIST

Rate of Convergence towards Hartree Dynamics with Singular Interaction Potential

We consider a system of N-Bosons with a two-body interaction potential $V \in L^2(\mathbb{R}^3) + L^\infty(\mathbb{R}^3)$, possibly more singular than the Coulomb interaction. We show that, with $H^1(\mathbb{R}^3)$ initial data, the difference between the many-body Schrödinger evolution in the mean-field regime and the corresponding Hartree dynamics is of order $1/N$, for any fixed time. The N-dependence of the bound is optimal.

MATTHIAS LIENERT, University of Tuebingen

A new kind of interacting quantum dynamics via integral equations for a relativistic two-particle wave function

In this talk, I present a new kind of interacting Lorentz invariant quantum dynamics which can be formulated using integral equations for a relativistic two-particle wave function $\psi(x_1, x_2)$ depending on two spacetime points $x_1, x_2 \in \mathbb{R}^4$. This idea is related to the Bethe-Salpeter equation of quantum field theory (but not identical to it). Such integral equations make it possible to express direct interactions with time delay at the quantum level. In the retarded case, the integral equations attain a Volterra-type structure and we were able to prove the existence and uniqueness of solutions for simple models.
This is joint work with Roderich Tumulka.

CAO LINGLING, Ecole des Ponts et Chaussées

The reduced Hartree-Fock model for extended defects in a Fermi sea

Studying the defects in materials remains an important subject in condensed matter physics. Localized defects have been well understood in insulators [Cancès et al, 2008][Lahbabi, 2014]. In this article, we establish a new mean-field model to study the extended defects in metals (modeled by the Fermi sea), where electron-electron interactions are separately modeled by Yukawa and Coulomb interactions. These extended defects typically correspond to taking out a slab of finite width in a three-dimensional perfect material. We work under the framework of reduced Hartree-Fock description [Solovej, 1991], i.e., the Hartree-Fock model without the exchange term. Inspired by the techniques developed in [Frank et al., 2013] in order to study perturbations in a Fermi sea, and in [Hainzl et al., 2005 a, b, 2007, 2009] for the purpose of studying the mean-field perturbations of Dirac sea in quantum electrodynamics, we justify the model by showing the existence of minimizers, and the convergence of Yukawa minimizers (resp. energy) to Coulomb minimizers (resp. energy) when Yukawa parameter tends to zero. Furthermore, we give the characterization of Yukawa minimizers. Finally, we give numerical illustrations of Friedel oscillations [Friedel, 1952], which are oscillations due to the screening of impurities for quantum many-body fermionic systems. We show that the model-generated Friedel oscillation coincides with the theoretic prediction of Friedel oscillation formula.

GIOVANNA MARCELLI, Universität Tübingen

Spin conductance and spin conductivity in topological insulators: analysis of Kubo-like terms

We investigate spin transport in 2-dimensional insulators, with the long-term goal of establishing whether any of the transport coefficients corresponds to the Fu-Kane-Mele index which characterizes $2d$ time-reversal-symmetric topological insulators. Inspired by the Kubo theory of charge transport, and by using a proper definition of the spin current operator [Phys. Rev. Lett. 96, 076604 (2006)], we define the Kubo-like spin conductance $G_K^{s_z}$ and spin conductivity $\sigma_K^{s_z}$. We prove that for any gapped, periodic, near-sighted discrete Hamiltonian, the above quantities are mathematically well-defined and the equality $G_K^{s_z} = \sigma_K^{s_z}$ holds true. Moreover, we argue that the physically relevant condition to obtain the equality above is the vanishing of the mesoscopic average of the spin-torque response, which holds true under our hypotheses on the Hamiltonian operator. This vanishing condition might be relevant in view of further extensions of the result, e.g. to ergodic random discrete Hamiltonians or to Schrödinger operators on the continuum. A central role in the proof is played by the trace per unit volume and by two generalizations of the trace, the principal value trace and its directional version. This talk is based on joint work with Gianluca Panati (La Sapienza, Roma) and Clément Tauber (ETH, Zürich).

JORDAN MOODIE, University of Birmingham

An exact power series representation of the Baker-Campbell-Hausdorff formula

The Baker-Campbell-Hausdorff formula is well known and given by $Z = \log(e^X e^Y) = X + Y + \frac{1}{2}[X,Y] + \frac{1}{12}[X,[X,Y] + \frac{1}{12}[Y,[Y,X]] + \cdots$, where it is not obvious what the dots represent. Considering the symmetric form of this formula, namely $S(A,B) = \log(e^{A/2}e^B e^{A/2})$, we find an exact power series representation in the matrix B. We find closed form A-dependent coefficients in the form of hyperbolic functions for all orders of B. Each of these coefficients represent an infinite number of terms in the original expansion, making truncation of the series much more controllable for small B but arbitrary A.

SIMONE MURRO, University of Freiburg

Invariant states on Weyl algebras for the action of the symplectic group

For any number h such that $\hbar := h/2\pi$ is irrational, let $\mathcal{A}_g^h$ be the corresponding Weyl $*$-algebra over $\mathbb{Z}^{2g}$ and consider the ergodic group of $*$-automorphisms of $\mathcal{A}_g^h$ induced by the action of $Sp(2g,\mathbb{Z})$ on $\mathbb{Z}^{2g}$. We show that the only $Sp(2g,\mathbb{Z})$-invariant state on $\mathcal{A}_g^h$ is the trace state τ.

HIROAKI NIIKUNI, Maebashi Institute of Technology

On the eigenvalue embedded in the spectral bands of Schrödinger operators on carbon nanotubes with impurities

In this talk, we discuss the spectra of Schrödinger operators on carbon nanotubes with impurities from the point of view of the theory of quantum graphs. In the case of carbon nanotubes without impurities, it is known that the spectrum has the band-gap structure, namely, the spectrum consists of infinitely many closed interval (spectral bands) and the flat bands (the set of eigenvalues with infinite multiplicities). In this talk, we give a finite number of impurities expressed as the δ vertex conditions to the operator. As a result, we obtain additional eigenvalues embedded in the spectral bands (not in the spectral gaps!). Furthermore, we have an estimate from below of the number of embedded eigenvalues in each spectral bands for a suitable strength of δ vertex conditions.

In this talk, we consider the case where impurities are symmetric with respect to z-axis and rotation. Due to the rotational symmetry, we obtain a unitary equivalence between our operator and the direct sum of a finite number of Schrödinger operators on the degenerate carbon nanotube. Furthermore, we utilize the space-symmetry on z-axis and decompose those operators as the direct sum of the reducted operators on half size degenerate carbon nanotube with the Dirichlet and Newmann boundary condition. After those decomposition, we examine the estimate from below of the number of eigenvalues in the spectral gaps of each reduced operator. Finally, we show that those eigenvalues are embedded in the spectral bands of other reduced operators.

MARKUS NÖTH, Ludwig-Maximilian University of Munich

On a Modern Variation of the Classic Results of Shale-Stinespring and Ruijsenaar on External Field QED

The classic results of Shale-Stinespring and Ruijsenaar together assert that an implementation of the time evolution of external field quantum electrodynamics between constant time hyperplanes on standard Fock space is possible if and only if the vector part of the external potential is zero. A modern approach by Deckert et al. to this problem achieves an implementation of time evolution for arbitrary external potentials by varying Fock spaces with time.

The residual freedom of this approach is discussed and classified. Furthermore, an attempt is made towards quantifying the speed of convergence of the relevant series.

JAN NOVÁK, Technical University in Liberec

One mathematical problem in non-relativistic quantum field theories

We will consider a QFT of single scalar field $\phi(t, \vec{x})$ in d spatial dimensions. We will work with polynomial shift transformations of the field and we will solve the task to find Lagrangian terms, which are invariant - up to total derivative - under these polynomial shift transformations. We will develop graphical representation and we prove that a superposition of an exact P_E-invariant with the superposition of Q minimal loopless 1-invariants results in a P-invariant, provided $P_E + 2Q \geq P$. We prove that the above result captures all P-invariants. Our result should deepen our knowledge about the study of naturalness in non-relativistic QFT's of the Lifshitz type.

NICOLA RENDELL, University of York

Infrared divergences for quantum fields in cosmological spacetimes

Our universe is believed to have experienced an inflationary period in its early stages of development. It is therefore of interest to understand the behaviour of the graviton two point function in an inflationary spacetime. We consider a background Friedman-LemaitreRobertson-Walker (FLRW) spacetime, which is a (slow roll) inflationary spacetime. The graviton two point function is known to be infrared divergent in such a spacetime. It has been previously found, in de Sitter spacetime, that a large coordinate gauge transformation can be used to remove the infrared divergence, and this suggests that the divergence should not lead to local physical effects. The next natural step appeared to be to see if this type of transformation could be used effectively in a FLRW spacetime. We found that this was indeed the case, and removed the leading order divergence of the graviton two point function. The correspondence between the gauge invariant part of the graviton two point function and the linearized Weyl tensor correlator gives a bound on the extent to which the divergence can be removed. This bound suggests that our transformation removes all unphysical infrared divergence, without affecting the physical behaviour of the two point function.

JULIEN RICAUD, Mathematisches Institut der LMU

Symmetry breaking in the periodic Thomas–Fermi–Dirac–von Weizsäcker model.

In this talk I will present a work on the Thomas–Fermi–Dirac–von Weizsäcker for a system composed of infinitely many nuclei placed on a periodic lattice and infinitely many electrons with a periodic density. The results are that if the Dirac constant is small enough then the electrons have the same periodicity as the nuclei and that, on the other hand, if the Dirac constant is large enough then the 2-periodic electronic minimizer is not 1-periodic, hence symmetry breaking occurs. We analyze in detail the behavior of the electrons when the Dirac constant tends to infinity and show that the electrons all concentrate around exactly one of the 8 nuclei of the unit cell of size 2, which is the explanation of the breaking of symmetry. Zooming at this point, the electronic density solves an effective nonlinear Schrödinger equation in the whole space with nonlinearity $u^{7/3} - u^{4/3}$. Our results rely on the analysis of this nonlinear equation, in particular on the uniqueness and non-degeneracy of positive solutions.

DANIEL SHEINBAUM, University of Biritsh Columbia

Adiabatic stability of Fermi surfaces and K-theory

I will present a classification of Fermi surfaces of non-interacting, discrete translation-invariant systems from electronic band theory, adiabatic evolution and their topological interpretations. For systems on a half-space and with a gapped bulk, this derivation naturally yields a K-theory classification. Given the $d-1$-dimensional surface Brillouin zone X_s of a d-dimensional half-space, this result implies that different classes of globally stable Fermi surfaces belong in $K^{-1}(X_s)$. I will also mention how to include symmetries through equivariant methods. This is based on joint work with A.Adem, O.Antolín-Camarena and G.W. Semenoff.

GERARDO BARRERA VARGAS, University of Alberta
On the cut-off phenomenon for hyperbolic attractors

Consider an ordinary differential equation with a fixed point that is a global attractor. Without loss of generality, assume that the fixed point is the origin. Under general conditions, at times goes by any solution of this equation approaches the fixed point exponentially fast. Now add a small random perturbation to this equation. It is well known that, again under very general conditions, as times goes by the solution of this stochastic equation converges to an equilibrium distribution that is well approximated by a Gaussian random variable of variance proportional to the strength of the perturbation. General theory of stochastic processes allows to show that this convergence, for each fixed perturbation, is again exponentially fast. We show that the convergence is actually abrupt: in a time windows of small size compared to the natural time scale of of the process, the distance to equilibrium drops from its maximal possible value to near zero, and only after this time window the convergence is exponentially fast. This is what is known as the cut-off phenomenon in the context of Markov chain of increasing complexity. Under a proper time scaling, we are able to prove convergence of the distance to equilibrium to a universal function, a fact known as profile cut-off. Joint work with Milton Jara.