Universal fluctuations in interacting dimers

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Based on joint works with V. Mastropietro and F. Toninelli

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Outline

1. Introduction and overview
2. Non-interacting dimers
3. Interacting dimers: main results

The scaling limit of the Gibbs measure of a critical stat-mech model is expected to be **universal**.

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Universality, scaling limits and Renormalization Group

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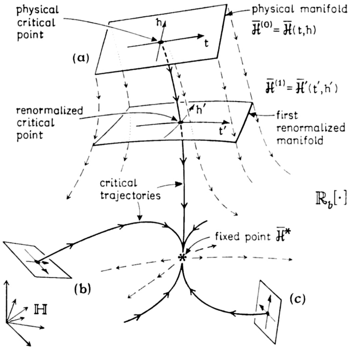
1. Integrate out the small-scale d.o.f., rescale, show that the critical model reaches a fixed point (Wilsonian RG).

2. Use CFT to classify the possible fixed points (complete classification in 2D; recent progress in 3D).



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1. Integrable models: Ising and dimers. Conformal invar. via discrete holomorphicity (Kenyon, Smirnov, Chelkak-Hongler--Izyurov, Dubedat, Duminil-Copin, ...) Universality: **geometric deformations** YES; **perturbations of Hamiltonian** NO

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2. Non-integrable models: interacting dimers, AT, 8V, 6V. Bulk scaling limit, via constructive RG (Mastropietro, Spencer, Giuliani, Falco, Benfatto, ...) Universality: **geometric deformations** NO; **perturbations of Hamiltonian** YES.

Dimers

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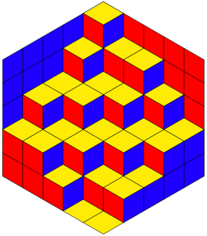

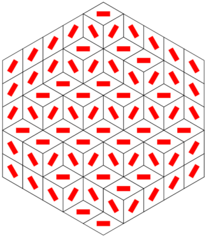

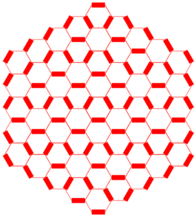

Note: the height describes a 3D Ising interface with tilted Dobrushin b.c.

At close-packing: family of solvable dimer models.
The partition function has a determinant structure.

Non-interacting dimers: exact solution and effective theory

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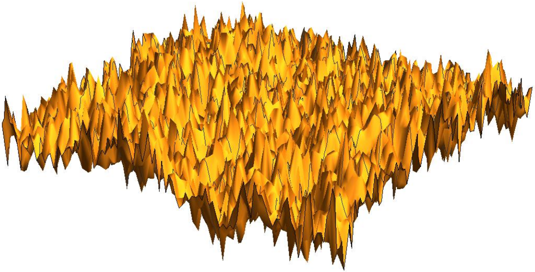

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NB: this proves the existence of a rough phase in 3D Ising at $T = 0$ with tilted Dobrushin b.c.

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Variance of GFF independent of slope $\rightsquigarrow$ Universality

Interacting dimers: main results (in brief)

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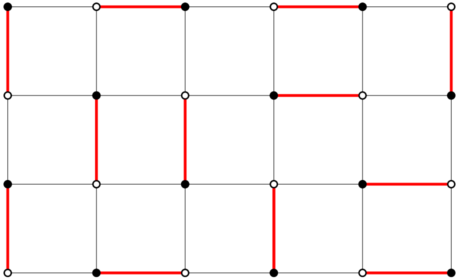
Subtle form of **universality**: the (pre-factor of the) variance equals the anomalous critical exponent of the dimer correlations $\Rightarrow$ Haldane relation.

Outline

Dimers and height function



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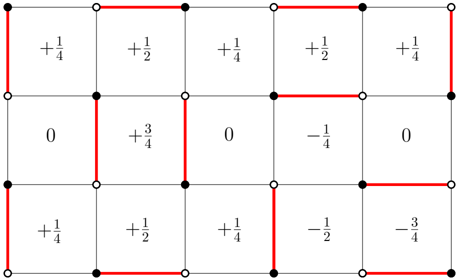

Height function:

$$h(f') - h(f) = \sum_{b \in C_{f \to f'}} \sigma_b(\mathbb{1}_b - 1/4)$$

$\sigma_b = \pm 1$ if b crossed with white on the right/left.

Non-interacting dimer model

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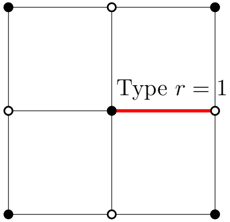

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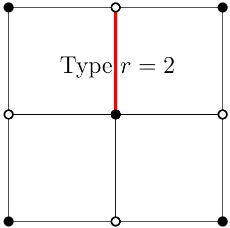

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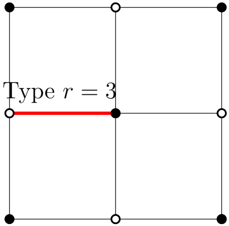

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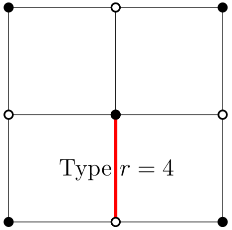

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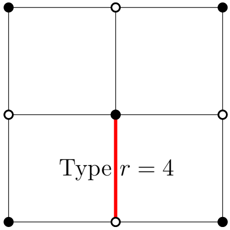

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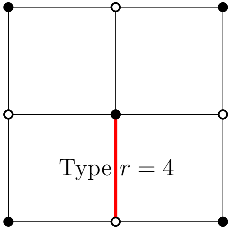

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$$\langle h(f + e_i) - h(f) \rangle_0 = \rho_i(t_1, t_2, t_3), \quad i = 1, 2.$$

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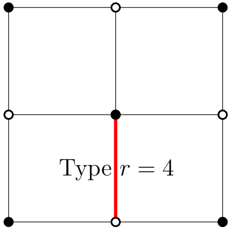

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The model is exactly solvable, e.g.,

$$Z_L^0 = \det K(\underline{t}), \quad \text{with} \quad K(\underline{t}) = \text{Kasteleyn matrix}.$$

Non-interacting dimer correlations

Non-interacting dimer-dimer correlations can be computed exactly (Kasteleyn, Temperley-Fisher):

$$\langle \mathbb{1}_{b(x,1)}; \mathbb{1}_{b(y,1)} \rangle_0 = -t_1^2 \, K^{-1}(x,y) \, K^{-1}(y,x),$$

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'Generically': two non-degenerate zeros $\Rightarrow K^{-1}(x,y)$ decays as $(dist.)^{-1}$: the system is critical.

Non-interacting height fluctuations

Height fluctuations grow logarithmically:

$$\langle h(f) - h(f'); h(f) - h(f') \rangle_0 \simeq \frac{1}{\pi^2} \log |f - f'|$$

as $|f - f'| \to \infty$ (Kenyon, Kenyon-Okounkov-Sheffield).

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The computation is based on: the definition

$$\langle h(f) - h(f'); h(f) - h(f') \rangle_0 = \sum_{b, b' \in C_{f \to f'}} \sigma_b \sigma_{b'} \langle \mathbb{1}_b; \mathbb{1}_{b'} \rangle_0,$$

the formula for $\langle \mathbb{1}_b; \mathbb{1}_{b'} \rangle_0$, and the path-indep. of the height.

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Building upon this (Kenyon):

- height fluctuations converge to massless **GFF**
- scaling limit is **conformally covariant**

Outline

Interacting model:

$$Z_L^{\lambda} = \sum_{D \in \mathcal{D}_L} \Big(\prod_{b \in D} t_{r(b)} \Big) e^{\lambda \sum_{x \in \Lambda} f(\tau_x D)},$$

where: λ is small, f is a local function of the dimer configuration around the origin, τ_x translates by x.

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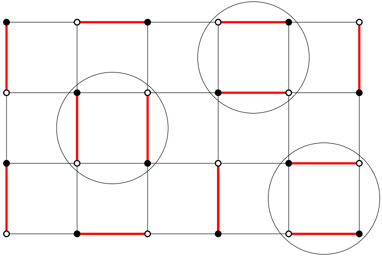

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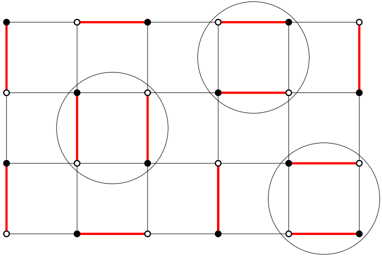

NB: for suitable f, the model reduces to 6V.

Generically, the model is non-integrable.

Our results don't depend on specific choice of f.

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Let t_1, t_2, t_3 be s.t. $\mu(k)$ has two distinct non-degen. zeros, $p^0_\pm$

(non-degenerate $\Leftrightarrow \alpha^0_\omega = \partial_{k_1}\mu(p^0_\omega)$ and $\beta^0_\omega = \partial_{k_2}\mu(p^0_\omega)$ are not parallel).

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Remarks

Proof $\Rightarrow$ algorithm for computing $K^{\lambda}_{\omega,r}, H^{\lambda}_{\omega,r}, \ldots$
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Use formula for $\langle \mathbb{1}_b; \mathbb{1}_{b'} \rangle_\lambda$ in that for height variance:

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it is not obvious that the growth is still logarithmic:
a priori, it may depend on the critical exp. $\nu(\lambda)$.

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$$\boxed{A(\lambda) = \nu(\lambda)}$$

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A and ν given by different renormalized expansions. No hope of showing $A = \nu$ from diagrammatics.

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Previous examples: solvable models (Luttinger, XXZ) and non-integrable variants (Benfatto-Mastropietro).

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After coarse-graining and rescaling,

$$h(f) \quad \xrightarrow{\quad d \quad} \quad \phi(x)$$

where ϕ is the massless GFF of covariance

$$\mathbb{E}(\phi(x)\phi(y)) = -\frac{A(\lambda)}{2\pi^2} \log |x - y|.$$

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Related results in random surface models: log fluctuations and roughening trans. in: anharmonic crystals, SOS model, 6V, Ginzburg-Landau type models (Brascamp-Lieb-Lebowitz, Fröhlich-Spencer, Falco, Ioffe-Shlosman-Velenik, Milos-Peled, Conlon-Spencer, Naddaf-Spencer, Giacomin-Olla-Spohn, Miller, ...)

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- Related results, via similar methods, for: Ashkin-Teller, $8V$, $6V$, XXZ, non-planar Ising.

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Thank you!