Aspects of Symmetries and RG Flow Constraints

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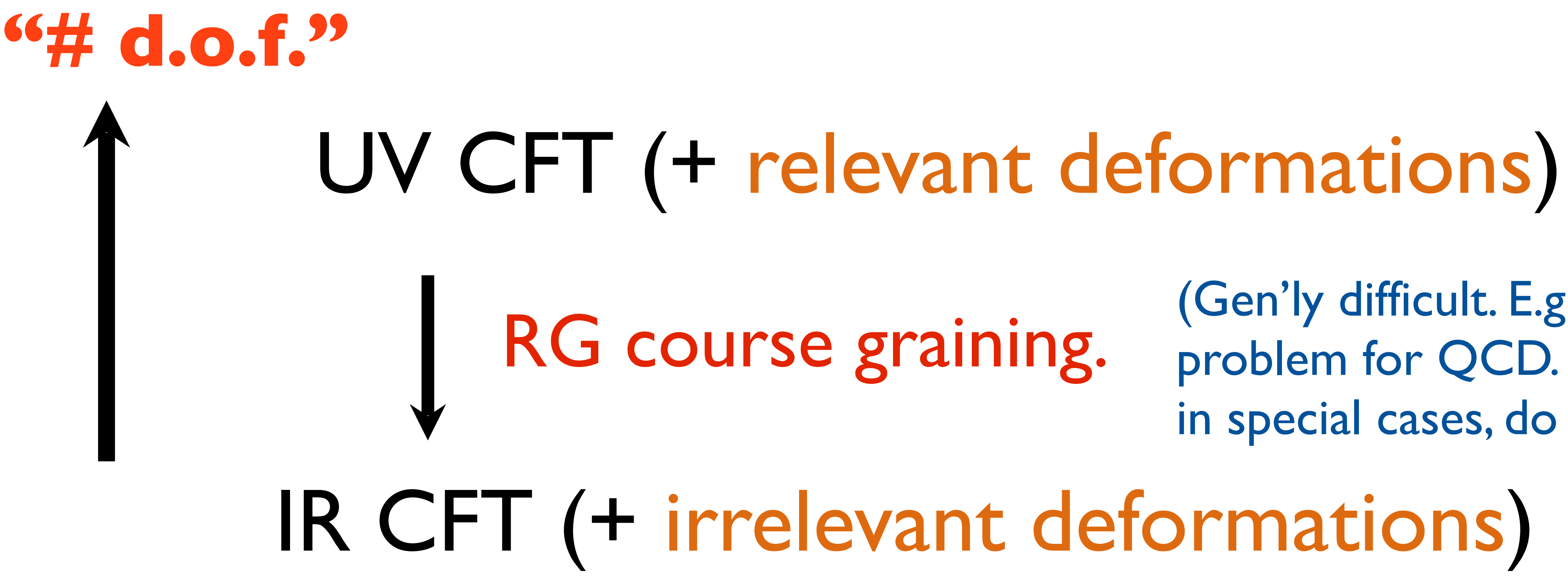
Thank the organizers for the opportunity to attend this conference and give this talk.

Based on work with Clay Cordova (IAS / U. Chicago) and Thomas Dumitrescu (UCLA), esp. 1802.04790

RG flows



“# d.o.f.”

UV CFT (+ relevant deformations)

RG course graining.

IR CFT (+ irrelevant deformations)

(Gen’ly difficult. E.g. open Clay Prize problem for QCD. Can instead **guess** in special cases, do non-trivial checks.)

- $$“\delta\mathcal{L}” = \sum_i g_i \mathcal{O}_i$$ (OK even if SCFT is non-Lagrangian)
- Move on the moduli space of (susy) vacua.
- Gauge a (e.g. UV or IR free) global symmetry.

RG flow constraints

- ’t Hooft **anomaly** matching for global symmetries + gravity. They must be constant on RG flows; match at endpoints.

- Reducing # of d.o.f. intuition. For d=2,4 (& d=6 susy) : a-theorem

$a_{\rm UV} \geq a_{\rm IR}$ $a \geq 0$ For unitary thys

conformal **anomaly**: $\langle T^\mu_\mu\rangle \sim aE_d + \sum_i c_i I_i$

★

a-theorem proof of Komargodski + Schwimmer via **conf’l anomaly matching.**

(d=odd: via sphere partition function / entanglement entropy.)

- Additional power from supersymmetry. Supermultiplets and supermultiplets of **anomalies**.

q-form global currents

- Conserved flavor current: $\partial^\mu J_\mu^a = 0$. Source: A_μ^a bkgd.

  = "q=0-form" global symmetry. (a = g Lie alg. index) $\delta A_\mu^a = (D_\mu \lambda)^a$

- Conserved **higher q-form global symmms:**

  Gaiotto, Kapustin, Seiberg, Willett and refs therein.

$j^{(q+1)}_{[\mu_1 \ldots \mu_{q+1}]}$ with $\partial^{\mu_1} j^{(q+1)}_{[\mu_1 \ldots \mu_{q+1}]} = 0.$ I.e. $d * j^{(q+1)} = 0$

$$Q(\Sigma_{d-q-1}) = \int_{\Sigma_{d-q-1}} * j^{(q+1)}$$

q>0: only abelian, U(1)$^{(q)}$ or discrete subgp.

$$\Delta_{\text{exact}}(j^{q+1}) = d - q - 1$$

Is q>0 possible for (S)CFTs? Often, "no". E.g. we show that no q>0 conserved current multiplets for 6d unitary SCFTs.

Couple all currents to background fields

- Poincare': Source = bkgd metric $g_{\mu\nu} = \delta_{ab} e^a_\mu e^b_\nu$

$$\delta e^{(1)a} = -\theta^{(0)a}_b e^{(1)b}$$

- Conserved flavor current: $\partial^\mu J^a_\mu = 0$. Source: A^a_μ bkgd.

Invariance: $\delta A^a_\mu = (D_\mu \lambda)^a$

- Conserved q>0 current:

$$S \supset \int B^{\mu_1 \ldots \mu_{q+1}} j_{[\mu_1 \ldots \mu_{q+1}]} dV = \int B^{(q+1)} \wedge \star j^{(q+1)}$$

$\delta B^{(q+1)} = d\Lambda^q$ invariance since $d \star j^{(q+1)} = 0$

Background gauge invariance encodes conservation laws.

Recall various anomalies

Effective action as fn of background fields:

$$W[\mathcal{B}] = -\log(\int [d\psi][dA]e^{-S[\mathcal{B},\psi,A]/\hbar})$$

$$W[\mathcal{B}+\delta\mathcal{B}] - W[\mathcal{B}] = \mathcal{A}[\mathcal{B}] = 2\pi i \int \mathcal{I}^{(d)}[\mathcal{B},\delta\mathcal{B}]$$

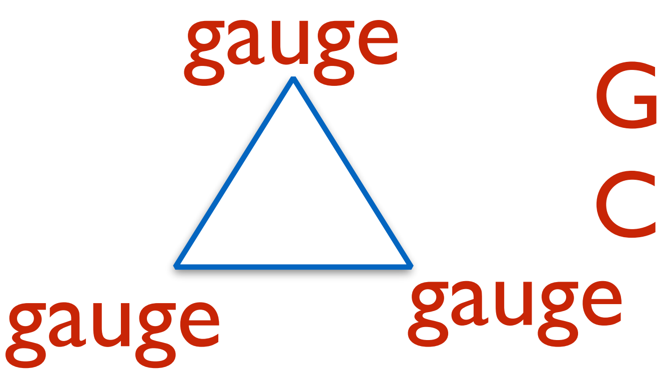
(descent procedure)

$$d\mathcal{I}^{(d)}[\mathcal{B},\delta\mathcal{B}] = \delta\mathcal{I}^{(d+1)}[\mathcal{B}], \;\; d\mathcal{I}^{(d+1)}[\mathcal{B}] = \mathcal{I}^{(d+2)}[\mathcal{B}]$$

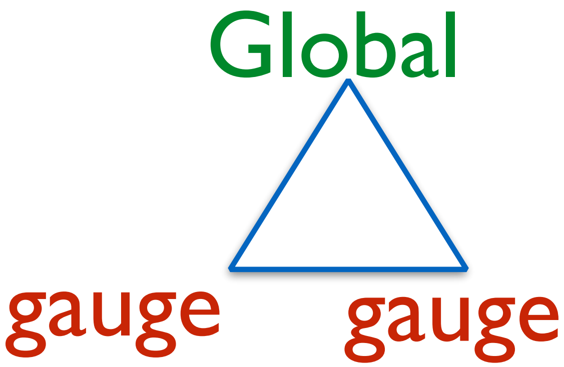
For d=2n, the matter content must be gauge anomaly free. Anomalies encoded in a topological d+2 form in gauge and global background field strength Chern classes, and Pontryagin classes for the background metric curvature. Compute via (n+1)-gon diagram, or inflow, etc. Calculable via various methods.

We discuss mixed gauge+ global anomalies. They quantum-deform the global symmetry group into a``2-group.''

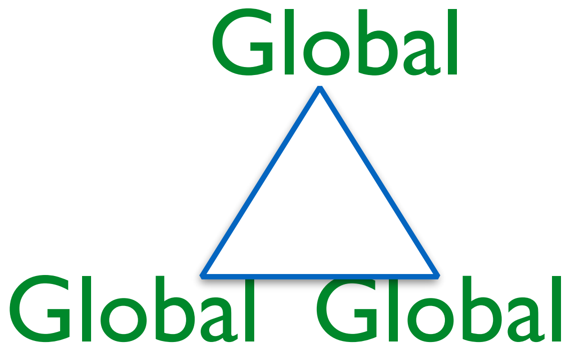
Anomalies (4d case)



Gauge anomalies must vanish for a healthy theory. Constrains chiral matter content.



ABJ anomaly, only for global U(1)s. If non-zero, global U(1) is just not a symm (explicitly broken by instantons, perhaps to a discrete subgp).



’t Hooft anomalies. **Useful** if non-zero: must be constant along RG flow, match at ends.

Our star:

Does not violate either symmetry. Deforms global symmetry to a 2-group symmetry.

$$d * j^{(1)}_{\mathrm{global}} = \frac{\kappa}{(2\pi)^2} F_{\mathrm{global}} \wedge F_{\mathrm{gauge}} = \frac{\kappa}{2\pi} F_{\mathrm{global}} \wedge * J_B^{(2)}$$

4d QED example

Consider a 4d (non-susy) QED, i.e. u(1) gauge theory, with N flavors of massless Dirac Fermion (IR free, needs a UV cutoff).

Global symmetry: $SU(N)_L^{(0)} \times SU(N)_R^{(0)} \times U(1)_B^{(1)}$

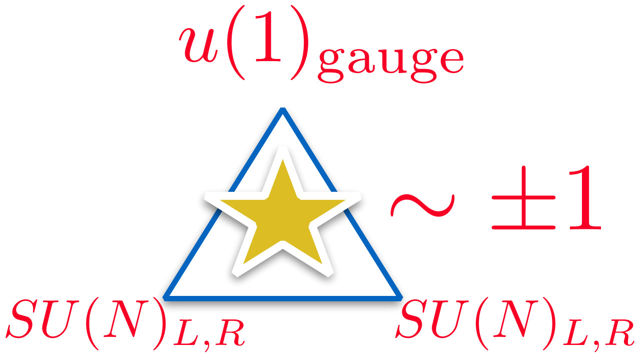
$U(1)_A^{(0)}$ broken by ABJ anomaly.

$U(1)_V^{(0)} \to u(1)_{\text{gauge}}$

$$j_B^{\mu\nu} \propto \epsilon^{\mu\nu\rho\sigma} f_{\rho\sigma}$$

global current

dyn. u(1) gauge field.



$\sim \pm 1$

Non-zero mixed anomaly. As we will discuss, it deforms the global symmetry to a 2-group symm.:

$$\left(SU(N)_{L-R}^{(0)} \times_{\kappa=1} U(1)_B^{(1)}\right) \times SU(N)_{L+R}^{(0)}$$

Chiral toy model examples

Consider a 4d (non-susy) theory with two 0-form flavor symms $U(1)_A$ and $U(1)_C$ and matter chiral Fermions with charges (q_A , q_C).

	q_A	q_C
ψ_1	1	3
ψ_2	1	4
ψ_3	-1	5
ψ_4	0	-6

$\kappa_{A^3} = \mathrm{Tr}U(1)_A^3 = 1$ ’t Hooft

$\kappa_{A^2C} = \mathrm{Tr}U(1)_A^2 U(1)_C = 12$ mixed

$\kappa_{AC^2} = \mathrm{Tr}U(1)_A U(1)_C^2 = 0$ ABJ=0

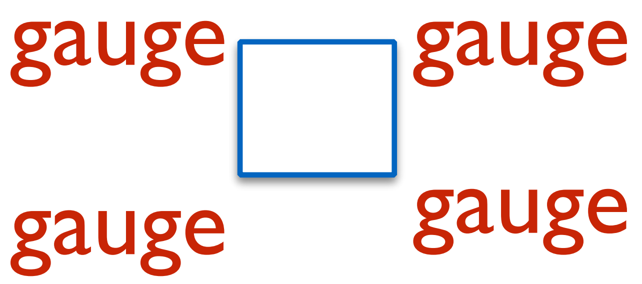
$\kappa_{C^3} = \mathrm{Tr}U(1)_C^3 = 0$ gauge=0

Take A=global and C=gauge symmetry. Non-zero ’t Hooft and mixed anomaly. $\mathcal{I}_6^{\text{mixed}} = (\kappa_{A^2C} c_2(F_A) + q_{C,tot} p_1(T)) \wedge c_1(f_c)$

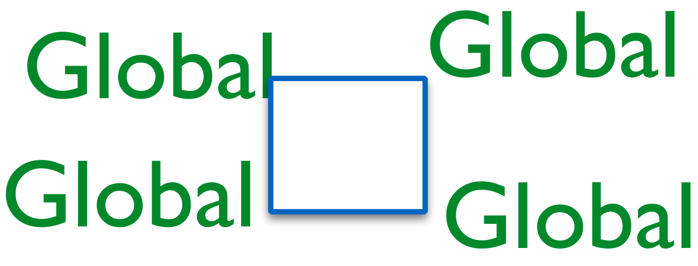
global

gauge

Likewise 6d anomalies



Gauge anomalies must vanish. Can use a dyn GSWS mechanism to cancel reducible parts.



’t Hooft anomalies. Useful if non-zero. Must be constant along RG flow, match at ends.

Does not violate any symmetry. Deforms global symmetry to a 2-group symmetry. Here the gauge group can be non-Abelian. (In 4d, there is only one gauge vertex, so it must be u(1).)

Example: small SO(32) instanton theory (Witten ’95)

$$\mathcal{I}_8^{\rm mixed} = c_2(F_{sp(N)})\left(c_2(F_{SO(32)}) + (16+N)p_1(T)\right)$$

6d anomalies (aside)

For 6d U(1) gauge theories, can also get a "4-group":

$$\mathcal{I} \supset k_{gGGG} c_1(f_{\text{gauge}}) c_3(F_{\text{Global}})$$

Does not violate any symmetry. Deforms global symmetry to a 4-group symmetry.

$$G_{\text{Global}} \times_\kappa U(1)^{(3)}_B$$

$$\delta A^{(1)}_{G_{\text{Global}}} = D\lambda^{(0)}_G$$

$$U(1)^{(3)}_B : \quad * j_4 = c_1(f_{\text{gauge}})$$

$$\delta B^{(4)} = d\Lambda^{(3)} + \frac{\kappa}{3!(2\pi)^2} Tr(\lambda_G F_G \wedge F_G)$$

$$\kappa \propto \kappa_{gGGG}$$

We will focus on the 2-group cases, i.e. involving 2-form bkgd gauge fields (can couple to strings).

Mixed gauge/global anomalies and 2-groups

$U(1)^{(1)}_B$ 4d: $\star j^{(2)} = c_1(f_{\text{gauge}}) = \frac{f_{gauge}}{2\pi}, \quad q_J = \int_{\Sigma_2} \star j^{(2)} \in \mathbf{Z}$

$U(1)^{(1)}_B$ 6d: $\star j^{(2)} = c_2(f_{\text{gauge}}) = \frac{1}{8\pi^2} \mathrm{Tr}\ \mathrm{f}_{\text{gauge}} \wedge \mathrm{f}_{\text{gauge}}, \quad \mathrm{q_J} = \int_{\Sigma_2} \star \mathrm{j}^{(2)} \in \mathbf{Z}$

Conserved since $d \star j^{(2)} = 0$, charged objects = e.g. ANO vortex strings (4d), instanton strings (6d).

Couple the 1-form global symmetries to 2-form background gauge fields B. $S_{4d,6d} \supset \int B \wedge \star j$

The mixed "anomaly" means that B shifts under a bkgd flavor or metric gauge transformation

$$A' = A + d\lambda_A,$$

$$B' = B + d\Lambda + \frac{\kappa}{2\pi} \lambda_A F_A$$

“2-group” global symmetry

If a non-trivial structure function interplay between a conserved q=2-form current and the other currents. Analogous to the Green-Schwarz mechanism for the global background fields coupled to the currents.

(See e.g. Kapustin and Thorngren papers, and refs therein.)

Global symmetry:
bkgd gauge transfs

$$G^{(0)} \times_{\hat{\kappa}} U(1)^{(1)}$$

$$\delta A^a_\mu = (D_\mu \lambda)^a \qquad \delta B^{(2)} = d\Lambda^{(1)} + \frac{\hat{\kappa}}{2\pi}\lambda^{(0)} dA^{(1)}$$

+ analog for Poincare’ SO(4) frame rotation of spin connection: $+\frac{\hat{\kappa}_{\mathcal{P}}}{16\pi}\mathrm{tr}(\theta^{(0)} d\omega^{(1)})$

$$H^{(3)} = dB^{(2)} - \frac{\hat{\kappa}_A}{2\pi} CS(A) - \frac{\hat{\kappa}_{\mathcal{P}}}{16\pi} CS(\omega),$$

dH sourced by background gauge & gravity instanton.

2-group structure constants

Global 0-form and 1-form symmetries: $G^{(0)}$ $G^{(1)}$

$\beta \in H^3(G^{(0)}, G^{(1)})$ we call them $\hat{\kappa}_{G^{(0)}}, \quad \hat{\kappa}_{\mathcal{P}}.$

Kapustin & Thorngren: Postnikov class. We also call them 2-group structure constants.

Coefficients of CS terms in invariant field strength $H^{(3)}$. For quantized charges, compact global groups, these coefficients must be integers: $\hat{\kappa}_{G^{(0)}}, \quad \hat{\kappa}_{\mathcal{P}} \in \mathbf{Z}$ They are scheme indep physical properties of the QFT. Can only arise at tree-level level or one-loop. Mixed anomaly terms give this symmetry.

E.g.: $U(1)_A^{(0)} u(1)_C^{(0)} \longrightarrow U(1)_A^{(0)} \times_{\hat{\kappa}_A, \hat{\kappa}_{\mathcal{P}}} U(1)_B^{(1)}$

GLOBAL

gauge

$$\hat{\kappa}_A = -\frac{1}{2}\kappa_{A^2C} \in \mathbf{Z}$$

$$\hat{\kappa}_{\mathcal{P}} = -\frac{1}{6}\kappa_{\mathcal{P}^2C} \in \mathbf{Z}$$

"Mixed anomaly" coeffs., so 2-group with no anomaly.

2-group affects reducible 't Hooft anomaly matching

E.g. $U(1)_A^{(0)} \times_{\hat{\kappa}_A} U(1)_B^{(1)}$ only has $\mathrm{Tr} U(1)_A^3$ 't Hooft anomaly matching mod $6\hat{\kappa}_A$, because of a possible counterterm: $S_{SG} = \frac{in}{2\pi}\int B^{(2)} \wedge F_A^{(2)}$, $U(1)_B^{(1)}$: $n \in \mathbf{Z}$

$\kappa_{A^3} \to \kappa_{A^3} + 6n\hat{\kappa}_A$ E.g. can gap if $\mathrm{Tr} U(1)_A^3 = 0 \bmod 6\hat{\kappa}_A$

TQFTs can give similar, but physical (non-counterterm) terms with fractional n. They can be used to match 't Hooft anomalies via a gapped TQFT. E.g. u(1)c gauge thy broken to Z_{q_C} TQFT by Higgs mechanism of field with charge $q_C > 1$. Allows $\mathrm{Tr} U(1)_A^3 \neq 0$ to be matched by gapped TQFT if $\mathrm{Tr} U(1)_A^3 = 0 \bmod 6n\hat{\kappa}_A, \quad q_C n \in \mathbf{Z}$

2-group vs CFT

Phrased in terms of Ward identities, contact term e.g.:

$$\frac{\partial}{\partial x_\mu}\langle j_\mu^A(x) j_\nu^A(y) J_{\rho\sigma}^B(x)\rangle = \frac{\hat{\kappa}_A}{2\pi}\partial^\lambda \delta^{(4)}(x-y)\langle J_{\nu\lambda}^B(y) J_{\rho\sigma}^B(z)\rangle$$

Implies a non-zero 3-point function also at separated points. Incompatible with additional constraints of CFT. Tension between 2-group vs CFT. 2-group can be an emergent symmetry, subject to constraints. E.g. the 4d u(1) gauge theory and 6d small SO(32) instanton examples are IR free, non-CFTs. Can UV complete if $U(1)_B$ is broken, accidental symmetry in IR. E.g. if u(1) is part of a non-Abelian UV completion, then $U(1)_B$ is broken (monopoles). Likewise for little string UV completion of small SO(32) instanton theory.

2-group RG flows, e.g.

$$\begin{tikzcd}[column sep=huge, row sep=huge]
\begin{array}{c} Z_1^{\mathrm{UV}}\left[A^{(1)}, C^{(1)}\right] \\ U(1)_A^{(0)} \times U(1)_C^{(0)} \end{array} \arrow[r, "\begin{array}{c} \int Dc^{(1)}\, Z_1^{\mathrm{UV}}\left[A^{(1)}, c^{(1)}\right] \\ \times \exp\left(\frac{i}{2\pi}\int B^{(2)} \wedge dc^{(1)}\right) \end{array}", "\text{gauge } U(1)_C^{(0)}"'] \arrow[d, "\text{RG Flow}"', "\begin{array}{l} \text{Anomaly} \\ \mathcal{A}_1 = \frac{i\kappa_{A^2C}}{8\pi^2}\int \lambda_A^{(0)}\, dA^{(1)} \wedge dC^{(1)} \\ \text{Matches} \end{array}"] & \begin{array}{c} Z_2^{\mathrm{UV}}\left[A^{(1)}, B^{(2)}\right] \\ U(1)_A^{(0)} \times_{\widehat{\kappa}_A} U(1)_B^{(1)} \end{array} \arrow[d, "\text{RG Flow}"', "\begin{array}{l} \text{Two-Group} \\ \text{Constant} \\ \widehat{\kappa}_A = -\frac{1}{2}\kappa_{A^2C} \end{array}"] \\
\begin{array}{c} Z_1^{\mathrm{IR}}\left[A^{(1)}, C^{(1)}\right] \\ U(1)_A^{(0)} \times U(1)_C^{(0)} \end{array} \arrow[r, "\text{gauge } U(1)_C^{(0)}", "\begin{array}{c} \int Dc^{(1)}\, Z_1^{\mathrm{IR}}\left[A^{(1)}, c^{(1)}\right] \\ \times \exp\left(\frac{i}{2\pi}\int B^{(2)} \wedge dc^{(1)}\right) \end{array}"'] & \begin{array}{c} Z_2^{\mathrm{IR}}\left[A^{(1)}, B^{(2)}\right] \\ U(1)_A^{(0)} \times_{\widehat{\kappa}_A} U(1)_B^{(1)} \end{array}
\end{tikzcd}$$

2-group vs CFT in d>4

1-form symmetry has conserved 2-form current, $\Delta[j_{\mu\nu}] = d-2$

Exists as a short rep of the conf'l gp, and for d>4 it is not necessarily free (it is co-closed, but not also closed as in 4d). Using conservation laws we show that, as in 4d

$\langle T^{\mu\nu}(x) T^{\kappa\lambda}(y) j^{\rho\sigma}(0)\rangle = 0$ So no 2-group with metric diffs.

But there were not enough constraints to rule out

$\langle J^{\mu}(x) J^{\nu}(y) j^{\rho\sigma}(z)\rangle \neq 0$ Possibly 2-group with global symms.

No 6d SCFT 2-group

Not even global $U(1)^{(1)}$(CDI): **no** unitary 6d SCFT reps contain global, conserved 2-form currents. So no 2-group symmetry nor mixed gauge, global anomalies can occur for 6d SCFTs. If it is a SCFT, any apparent such mixed anomalies must be cancelled by the GSWS mechanism, along with the reducible gauge anomalies. Justifies prescription given by **Ohmori, Shimizu, Tachikawa, and Yonekura.** This affects ’t Hooft anomaly coefficients for e.g. $SU(2)_R$ and in various examples with gauge multiplets it turns out to be crucial for ensuring positivity of the conf’l anomaly $\mathbf{a}_{\mathrm{SCFT}}$ computed via ’t Hooft anomalies.

a, for 6d SCFTs with gauge flds:

E.g. SU(N) gauge group, 2N flavors, 1 tensor + anomaly cancellation for reducible gauge + mixed gauge + R-symmetry anomalies. Use $a^{\text{origin}} = \frac{16}{7}(\underset{\uparrow}{\alpha} - \underset{\uparrow}{\beta} + \underset{\uparrow}{\gamma}) + \frac{6}{7}\underset{\uparrow}{\delta}$

(R-symmetry + diff. 't Hooft anomalies, CDI '15)

=0*(via GSWS)

V H T *AC **:GSWS**

$$a_{SCFT} = (N^2 - 1)(-\frac{251}{210}) + 2N^2(\frac{11}{210}) + \frac{199}{210} + \frac{96}{7}N^2 > 0.$$

NB, there cannot be a conserved 2-form current in SCFT at the origin, despite apparent $c_2(f_{\text{gauge}})$: it sources dH and is believed to become part of a (poorly understood) non-Abelian version so not gauge invariant current at the origin.

Also

- Study effect of gauging the 2-group.
- Study effects for strings and line defects.
- Work in progress on other applications in 4d and 6d.
- Thank you!