Random quantum correlations are generically non-classical

Joint work with Carlos González-Guillén, Carlos Palazuelos, Ignacio Villanueva

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ICMP Montréal, QI session - July 24th 2018

Two-player two-outcome non-local games

Two cooperating but separated players Alice & Bob. Each of them receives an input and has to produce an output, which makes them win or loose a given amount. To try and maximize their gain, they can agree on a strategy before the game starts but then cannot communicate anymore.

Questions : $i \in \{1, \ldots, n\}$ → A | $j \in \{1, \ldots, n\}$ → B w.p. $\Pi(ij)$

Answers : A → $x \in \{+, -\}$ | B → $y \in \{+, -\}$ w.p. $P(xy|ij)$

$$\text{A \& B gain } V(ijxy) = \begin{cases} +V(ij) \text{ if } x = y \\ -V(ij) \text{ if } x \neq y \end{cases}$$

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Given a strategy (i.e. a conditional p.d.) P for A & B, the associated correlation τ is the $n \times n$ matrix defined by :

$$\forall\, i,j \in \{1,\ldots,n\},\ \tau_{ij} = P(++|ij) + P(--|ij) - P(+-|ij) - P(-+|ij).$$

Goal of A & B : Maximize their expected gain, i.e. $\max \left\{ \sum_{i,j=1}^{n} \Pi(ij)V(ij)\tau_{ij},\ P \text{ allowed strategy} \right\}$.

Allowed strategies and associated correlation matrices

Strategies :

- **Classical strategy :** A & B share common randomness $\rightarrow P(xy|ij) = \sum_\lambda q_\lambda A(x|i\lambda)B(y|j\lambda)$,
  with $\{q_\lambda\}_\lambda, \{A(+|i\lambda), A(-|i\lambda)\}, \{B(+|j\lambda), B(-|j\lambda)\}$ p.d.'s.
- **Quantum strategy :** A & B share a bipartite quantum state $\rightarrow P(xy|ij) = \mathrm{Tr}(A_i^x \otimes B_j^y \rho)$,
  with ρ state on $\mathcal{H}_A \otimes \mathcal{H}_B$, $(A_i^+, A_i^-), (B_j^+, B_j^-)$ measurements on $\mathcal{H}_A, \mathcal{H}_B$.

[ρ *state* on $\mathcal{H}$: $\rho \geqslant 0$, $\mathrm{Tr}\,\rho = 1$. (C^+, C^-) *measurement* on $\mathcal{H}$: $C^+, C^- \geqslant 0$, $C^+ + C^- = \mathrm{Id}$.
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Correlations :

- **Classical correlation :** $\tau \in \mathcal{C} := \left\{ \left(\mathbf{E}[X_i Y_j] \right)_{1 \leqslant i,j \leqslant n},\ |X_i|, |Y_j| \leqslant 1 \text{ a.s.} \right\}$.
- **Quantum correlation :** $\tau \in \mathcal{Q} := \left\{ \left(\mathrm{Tr}[X_i \otimes Y_j \rho] \right)_{1 \leqslant i,j \leqslant n},\ \begin{cases} X_i^* = X_i,\ Y_j^* = Y_j \\ \|X_i\|_\infty, \|Y_j\|_\infty \leqslant 1 \end{cases} \rho \text{ state} \right\}$.

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Proposition [Characterization of $\mathcal{C}$ and $\mathcal{Q}$ (Tsirelson)]

$\mathcal{C} = \mathrm{conv}\left\{ (\alpha_i\beta_j)_{1\leqslant i,j\leqslant n},\ \alpha_i, \beta_j = \pm 1 \right\}$ and $\mathcal{Q} = \mathrm{conv}\left\{ (\langle u_i, v_j\rangle)_{1\leqslant i,j\leqslant n},\ u_i, v_j \in S_{\mathbf{R}^m},\ m \in \mathbf{N} \right\}$

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Bell inequality violation : Quantum players may perform strictly better than classical ones, i.e. there exist Π and V s.t. $\max\left\{ \sum_{i,j} \Pi(ij) V(ij) \tau_{ij},\ \tau \in \mathcal{Q} \right\} > \max\left\{ \sum_{i,j} \Pi(ij) V(ij) \tau_{ij},\ \tau \in \mathcal{C} \right\}$.

Correlation matrices and tensor norms

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Definition/Proposition [The dual norms $\ell_1^n\otimes_\varepsilon\ell_1^n$ and $\ell_\infty^n\otimes_\pi\ell_\infty^n$ on $\mathbf{R}^n\otimes\mathbf{R}^n$]

Define the norm $\|\cdot\|_{\ell_1^n\otimes_\varepsilon\ell_1^n}$ by : $\|M\|_{\ell_1^n\otimes_\varepsilon\ell_1^n} := \sup\left\{\sum_{i,j=1}^n M_{ij}\alpha_i\beta_j,\ \alpha_i,\beta_j=\pm 1\right\}$.

Denote by $\|\cdot\|_{\ell_\infty^n\otimes_\pi\ell_\infty^n}$ its dual norm. $\left[\ \|\tau\|_{\ell_\infty^n\otimes_\pi\ell_\infty^n} := \inf\left\{\sum_{k=1}^N \|x_k\|_\infty\|y_k\|_\infty,\ \tau=\sum_{k=1}^N x_k\otimes y_k\right\}\ \right]$

Then : $\tau\in\mathcal{C}\ \Leftrightarrow\ \forall\, M$ s.t. $\|M\|_{\ell_1^n\otimes_\varepsilon\ell_1^n}\leqslant 1,\ \mathrm{Tr}(\tau M^t)\leqslant 1\ \Leftrightarrow\ \|\tau\|_{\ell_\infty^n\otimes_\pi\ell_\infty^n}\leqslant 1$.

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Definition/Proposition [The dual norms γ_2^* and γ_2 on $\mathbf{R}^n \otimes \mathbf{R}^n$]

Define the norm $\gamma_2^*(\cdot)$ by $\gamma_2^*(M) := \sup \left\{ \sum_{i,j=1}^n M_{ij}\langle u_i, v_j\rangle,\ u_i, v_j \in S_{\mathbf{R}^m},\ m \in \mathbf{N} \right\}$.

Denote by $\gamma_2(\cdot)$ its dual norm. $\left[\gamma_2(\tau) := \inf \left\{ \max_{1\leqslant i\leqslant n} \|R_i(X)\|_2 \max_{1\leqslant j\leqslant n} \|C_j(Y)\|_2,\ \tau = XY \right\} \right]$

Then : $\tau \in \mathcal{Q} \Leftrightarrow \forall\, M$ s.t. $\gamma_2^*(M) \leqslant 1,\ \mathrm{Tr}(\tau M^t) \leqslant 1 \Leftrightarrow \gamma_2(\tau) \leqslant 1$.

Tighter inequalities between "classical" and "quantum" norms on random inputs

Known : By Grothendieck's inequality (Grothendieck/Krivine), for any $n \times n$ matrix T,

$$\gamma_2(T) \leqslant \|T\|_{\ell_\infty^n \otimes_\pi \ell_\infty^n} \leqslant K_G \gamma_2(T), \text{ where } 1.67 < K_G < 1.79.$$

$\rightarrow$ No unbounded ratio (as n grows) between the "classical" and "quantum" norms of T.

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Theorem (González-Guillén/L./Palazuelos/Villanueva)

Let T be an $n \times n$ random matrix satisfying the two following assumptions : its distribution is bi-orthogonally invariant and w.h.p. $\|T\|_\infty \leqslant (r + o(1))\|T\|_1/n$. Then w.h.p.

$$\|T\|_{\ell_\infty^n \otimes_\pi \ell_\infty^n} \geqslant \left(\sqrt{\frac{16}{15}} - o(1)\right)\gamma_2(T) > \gamma_2(T).$$

Consequence : The $n \times n$ random correlation matrix $\tau = T/\gamma_2(T)$ is quantum (by construction) but w.h.p. non-classical.

Consequences of this result and main technical ingredients in its proof

Examples of applications :

- Let G be an $n \times n$ Gaussian matrix.
  $\tau = G/\gamma_2(G)$ is uniformly distributed on the border of $\mathcal{Q}$ but w.h.p. not in $\mathcal{C}$.

  $\rightarrow$ The borders of $\mathcal{C}$ and $\mathcal{Q}$ do not coincide in typical directions.

- Let $u_1, \ldots, u_n, v_1, \ldots, v_n$ be independent and uniformly distributed unit vectors in $\mathbf{R}^m$.
  $\tau = (\langle u_i, v_j \rangle)_{1 \leqslant i,j \leqslant n}$ is in $\mathcal{Q}$ but w.h.p. not in $\mathcal{C}$ if $m/n < 0.13$.

  $\rightarrow$ Bridging the gap between this result and the opposite one, stating that τ is w.h.p. in $\mathcal{C}$ if $m/n > 2$ (González-Guillén/Jiménez/Palazuelos/Villanueva) ?

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Two main technical lemmas needed in order to prove this result :

- SVD of a bi-orthogonally invariant random matrix T :
  $T \sim U \Sigma V^t$ with U, V, Σ independent, U, V uniformly distributed orthogonal matrices, Σ diagonal positive semidefinite matrix.

- Levy's lemma for an L-Lipschitz function $f : S_{\mathbf{R}^n} \rightarrow \mathbf{R}$ with median M_f (w.r.t. the uniform measure) :
  $\forall\, 0 < \theta < \pi/2,\ \mathbf{P}\left(f \gtrless M_f \pm (\cos\theta) L\right) \leqslant \frac{1}{2} (\sin\theta)^{n-1} \leqslant \frac{1}{2} e^{-(n-1)(\cos\theta)^2/2}.$

Two intermediate results

Proposition [Upper bounding the quantum norm of a random matrix]

Let T be an $n \times n$ random matrix s.t. its distribution is bi-orthogonally invariant and w.h.p. $\|T\|_\infty \leqslant (r+o(1))\|T\|_1/n$. Then w.h.p.

$$\gamma_2(T) \leqslant (1+o(1))\,\frac{\|T\|_1}{n}.$$

Remark : This result is optimal, i.e. we also have w.h.p. $\gamma_2(T) \geqslant (1-o(1))\,\frac{\|T\|_1}{n}$.

Proposition [Lower bounding the classical norm of a random matrix]

Let T be an $n \times n$ random matrix s.t. its distribution is bi-orthogonally invariant. Then w.h.p.

$$\|T\|_{\ell_\infty^n \otimes_\pi \ell_\infty^n} \geqslant \left(\sqrt{\frac{16}{15}} - o(1)\right)\frac{\|T\|_1}{n}.$$

Remark : This result is potentially non-optimal.

Indeed, it is proved by duality, i.e. by finding M s.t. w.h.p. $\frac{\mathrm{Tr}(TM^t)}{\|M\|_{\ell_1^n \otimes_\varepsilon \ell_1^n}} \geqslant \left(\sqrt{\frac{16}{15}} - o(1)\right)\frac{\|T\|_1}{n}$.

But the choice of M may not be the best and the upper bound on $\|M\|_{\ell_1^n \otimes_\varepsilon \ell_1^n}$ may not be tight...

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  Answer (Ambainis/Bačkurs/Balodis/Kravčenko/Ozols/Smotrovs/Virza) : If M is an $n \times n$ Gaussian or Bernoulli matrix, then w.h.p.

$$\gamma_2^*(M) \geqslant \left(\frac{1}{\sqrt{\ln 2}} - o(1) \right) \|M\|_{\ell_1^n \otimes_\varepsilon \ell_1^n} > \|M\|_{\ell_1^n \otimes_\varepsilon \ell_1^n}.$$

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- **Weaker corollaries :** Separations of $\mathcal{Q}^*$ vs $\mathcal{C}^*$ and $\mathcal{Q}$ vs $\mathcal{C}$ in terms of mean width w, i.e.

$$w(\mathcal{Q}^*) < w(\mathcal{C}^*) \text{ and } w(\mathcal{Q}) > w(\mathcal{C}).$$

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- What about the generic case in more general settings (more players, more outcomes) ? Basically nothing is known...

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