Toward a phase diagram of longitudinal string interactions
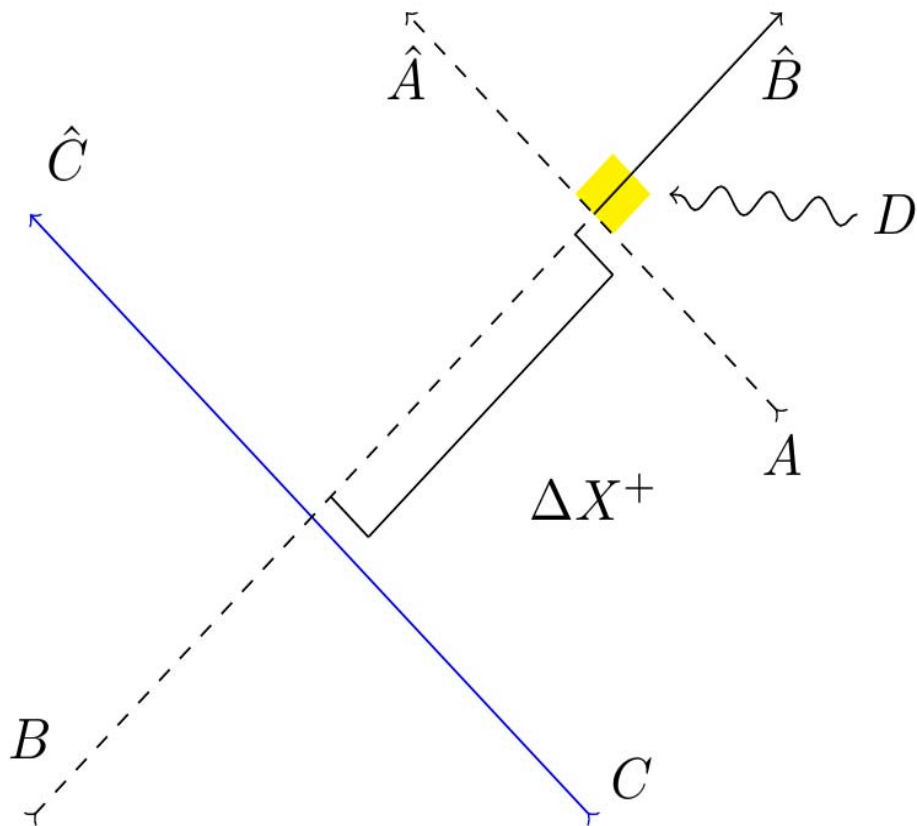
Based on:

M. Dodelson, ES Phys. Rev. D (2017)
MD, ES, G. Torroba Phys. Rev. D (2017)

D. Mathis, A. Mousatov and ES (works in progress)

Basic question: over what range in the longitudinal direction (i.e. the direction of relative motion) do strings interact?

$\hat{A}$

$\hat{B}$

$\hat{C}$

D

A

ΔX^+

B

C

String Spreading

Susskind '94 + Lowe Polchinski Thorlacius Uglum '95... (authors had different interpretations...)

Light Cone gauge $X^- \sim p^- \tau$,

Constraint determines X^+ in terms of $X^\perp$

$$\langle \psi | (X_\perp - x_\perp)^2 | \psi \rangle = \sum_n^{n_{max}} \frac{1}{n} = \log \frac{n_{max}}{n_0} + O\left(\frac{1}{n_{max}}\right)$$

$$\langle \psi | (X^+ - x^+)^2 | \psi \rangle \approx \frac{1}{(p^-)^2} \sum^{n_{max}} n \approx \frac{n_{max}^2}{(p^-)^2}$$

$$n_{max} \iff \text{light cone time resolution}$$

Predicts $\Delta X^+ \sim k^+_{det} \sim \frac{E}{k_\perp^2} \leftarrow$ if $> \frac{1}{g'}$

Two quadratically divergent sums appear in light cone string theory calculations. In spreading, the transverse Virasoro generators imply

$$\langle (X^+(\sigma, 0) - x^+)^2 \rangle \propto \frac{c_\perp}{(p^-)^2} \sum_n n \sim \frac{n_{\max}^2}{(p^-)^2}$$

On the other hand, in the mass shell formula we have a quadratically divergent sum which is subtracted (local counterterm tunes away the worldsheet c.c.):

$$\sum_{n>0} n$$

In the superstring, the two are cleanly separated: the quadratically divergent worldsheet cosmological constant cancels, but the central term in the X^+ modes (transverse Virasoro generators) does not cancel.

Light cone time resolution (refined prediction)

T detector

Source
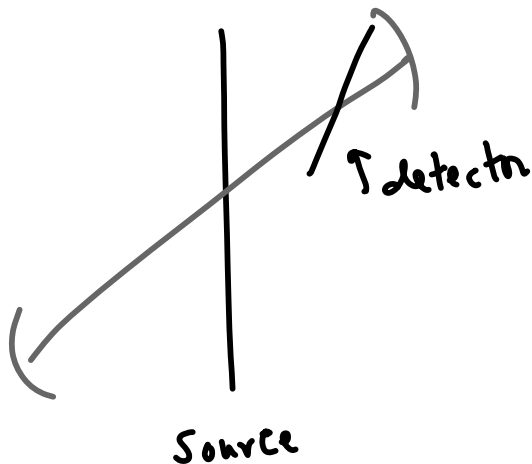
Note that this effect is causal, just non-local (the string is spread out, so can potentially interact before its center reaches the detector).

Measurement degrades
as $\uparrow P_\perp$: conservatively

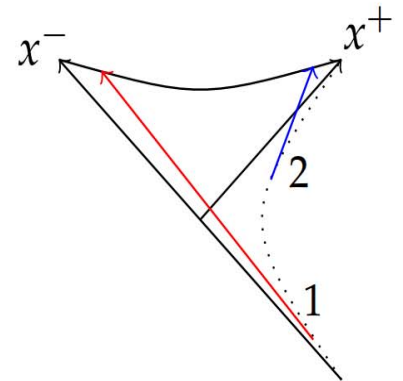$$\Delta X^- \sim \frac{P_\perp^2 + m^2}{P_{det}^+} \quad , \quad n_{max} \sim \frac{P_s^- P_d^+}{P_\perp^2 + m^2}$$

$$\Rightarrow \Delta X^+_{spr} \sim \frac{E_{det}}{P_\perp^2}$$
RMS

This n_max agrees precisely with BPST '06 2->2 calculation, and with Gross-Mende

One motivation: weakly curved geometries with a horizon: evolution of trajectories of (say) two probes sent in with modest energy leads to a large *nonlocal* invariant energy in the near horizon region.

$$ds^2 = -\left(1 - \frac{r_s}{r}\right) dt^2 + \left(1 - \frac{r_s}{r}\right)^{-1} dr^2$$
$$= -\frac{2r_s}{r}e^{1-r/r_s}dx^+ dx^-.$$

$$s_{early,late} = s_0 e^{\Delta t/2r_s}$$

How nonlocal is string theory in the longitudinal direction of relative motion between a string and a second object (detector)? String theory introduces beyond-GR physics at the singularity for obvious reasons, what about the horizon?

cf string production Bachas, McAllister/Mitra, Senatore et al, Polchinski, ES '14, Puhm, Rojas, Ugajin '16.

Black hole mining:

$\mu = \frac{1}{\alpha'}$

m

ΔX^+

①
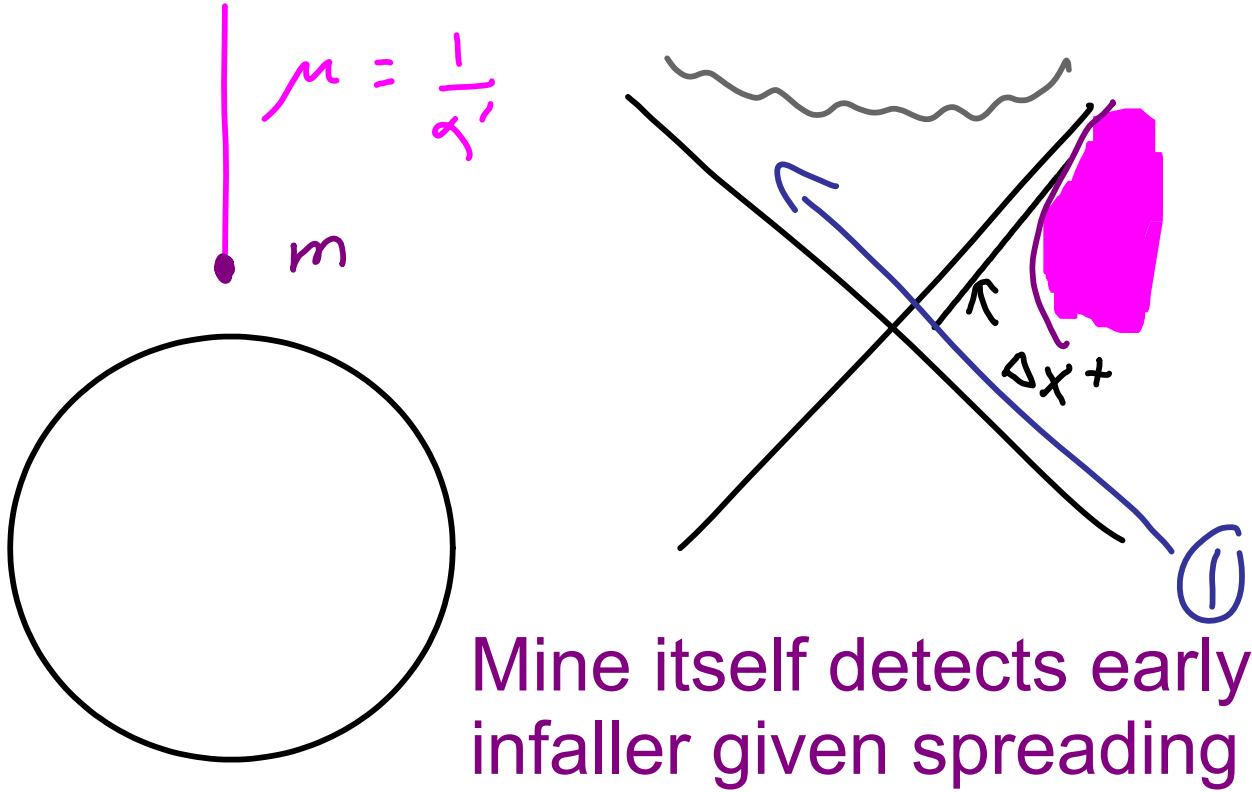
Mine itself detects early infaller given spreading

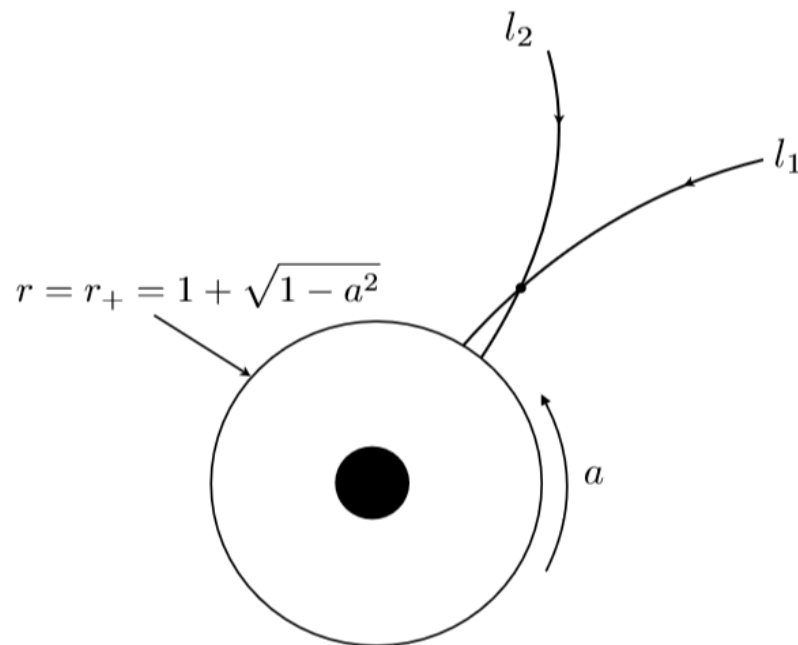$$E(R) = m\sqrt{g_{00}(R)} + \mu \int_R^{R_{UV}} dr\sqrt{g_{00}g_{rr}} = m\sqrt{1 - r_s/R} + \mu(R_{UV} - R)$$

$$E'(R) \equiv 0 \quad \Rightarrow \quad m = 2\frac{R^2\mu}{r_s}\sqrt{1 - r_s/R}$$

$$\frac{\Delta X_{spr}^+}{\Delta X^+} \sim \frac{p_{mine}^+}{m^2 X^+} = \frac{\alpha'}{4(R^2 - Rr_s)} = \mathcal{O}(1)$$

$$\Rightarrow L_{proper} \sim \sqrt{\alpha'}$$

Kerr black hole high energy interactions (Banados, Silk, West '09)

Large c.o.m. energy for very near-extremal case, for very near-horizon interactions. With string spreading, can de-tune the local collision.

$$r = r_+ = 1 + \sqrt{1 - a^2}$$

l_2

l_1

a

$$-2(1 + \sqrt{1 + a}) < l < 2(1 + \sqrt{1 - a})$$

Also AdS/CFT `glueball' scattering etc.

So it is worth understanding whether the physical (light cone gauge) prediction is correct.

Plan for the rest:

*Tree-level test
*Generalization to closed string (growing effect in time in BH)
*Quantum effects
*Building blocks for Eikonal amplitude
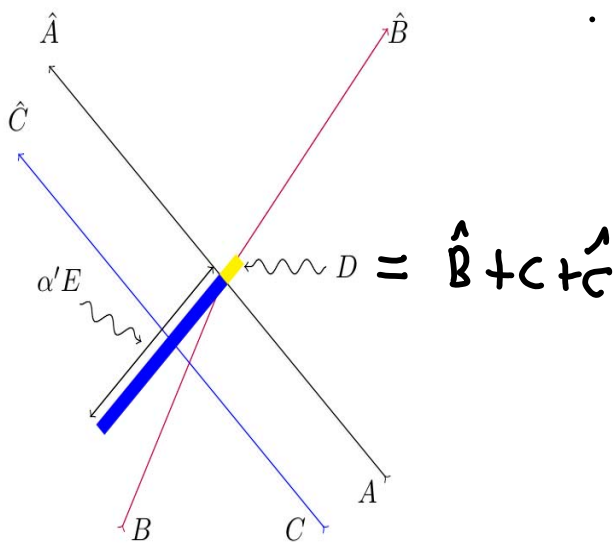*Scrambling timescale

A similar setup to the black hole appears at six points in flat space string scattering

Test of longitudinal interaction.

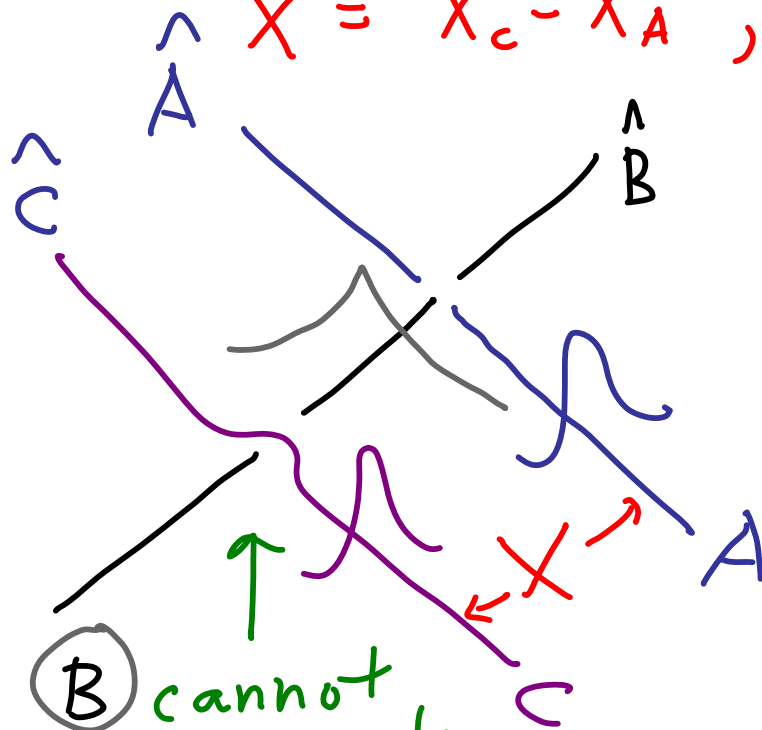$$g_s(X^+) = g_0 e^{V \cdot X} = g_0 e^{-V^- X^+}$$

$$D = \hat{B} + c + \hat{c}$$
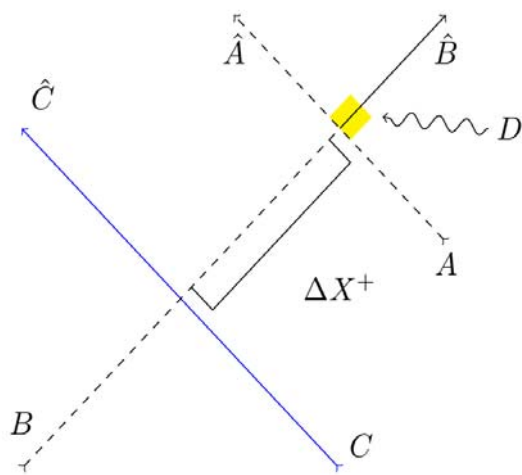
$$\Delta X^+_{\text{spreading}} \overset{?}{\sim} \frac{k_D^+}{k_D^+ k_D^-} \sim \frac{E_{\hat{B}}}{k_D^+ k_D^-}.$$

$$A(X) \equiv \int \Psi_{X,P_0}(\breve{p}) \mathcal{A}(\breve{p})$$

wavepacket peaked at

$X = X_C - X_A$, $p = p_0$

$\hat{A}$

$\hat{C}$

$\hat{B}$

$\hat{A}$

A

C

$\circledast{B}$ cannot interact directly by gauge invariance

Open string orderings $A\hat{A}\hat{C}C\hat{B}B$, $A\hat{A}B\hat{B}C\hat{C}$

We include wavepackets that strongly constrain the momenta in the longitudinal direction, while providing enough spatial resolution to test ~ $E\alpha'$ spread.

We focus on a small momentum range in which the amplitude exhibits a certain structure, finding a significant test of the effect there.

Tree-level QFT provides a control group, operationally: analyze with same wavepackets and check if the resulting amplitude $A(X)$ is parametrically suppressed compared to the string theory case.

Lightlike linear dilaton traces the location of interaction, consistent with scattering at spreading radius (rather than wavepacket tail).

QFT comparison model(s)

$$A_{\text{QFT1}} \sim \lambda_4^{(1)} \lambda_4^{(2)} \frac{1}{k_D^2 - i\epsilon} \delta^{(3)}(\sum \tilde{k}_I)$$
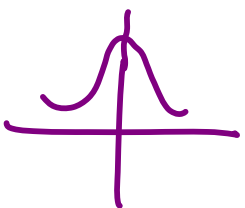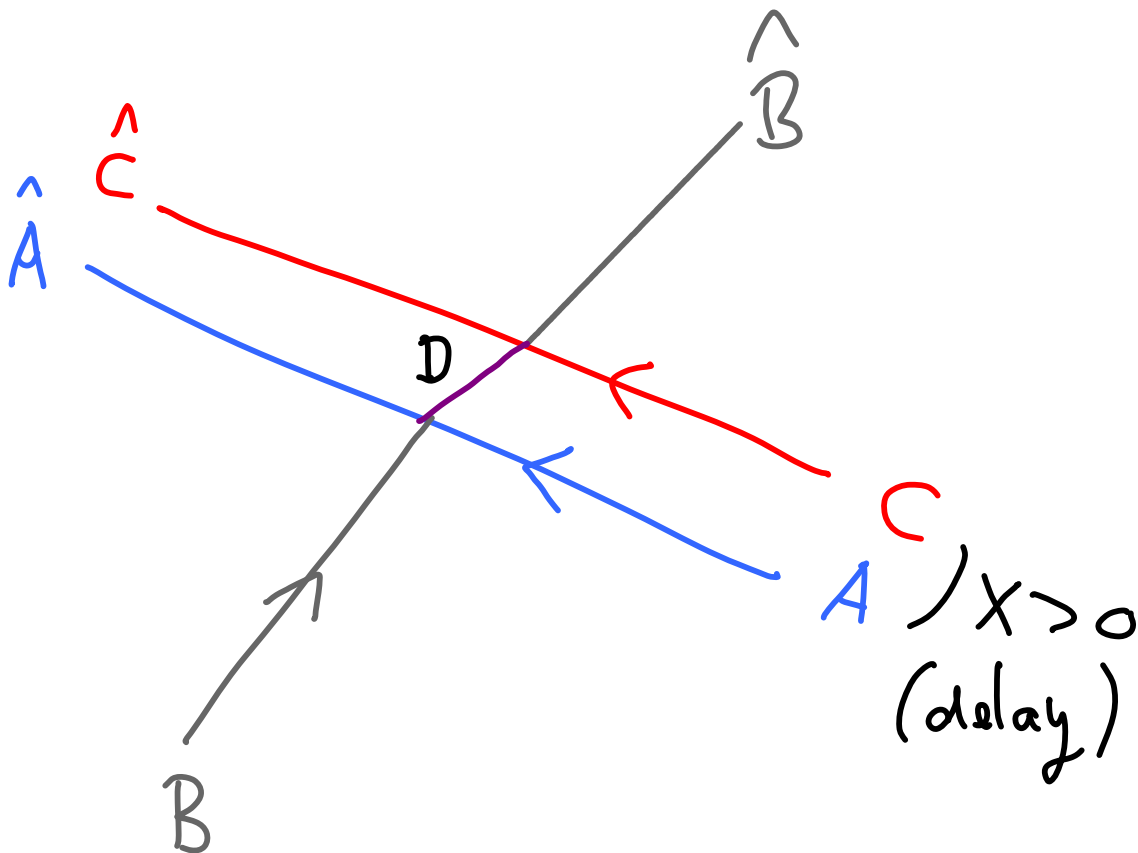
Basic result (delicate choice of Ψ):
A(X) in ST exhibits properties:
*Beats QFT at X ~-Eα'
*A(X) support peaked there
*string coupling g$_S$(X) consistent with
interaction at spreading radius.
*Amplitude not convergent sum of QFT
propagators (not sum of QFT1 models)

$$A^{(X)}_{QFT} \sim \frac{1}{k^2_{1,0}} \lambda^{(4)}_1 \lambda^{(4)}_2 \, e^{-X^2 \frac{\sigma^2}{2}}$$

$$A_{ST}(X) \sim 2^n A^{(4)}_1 A^{(4)}_2 \, e^{-\frac{(X+X_*)^2 \sigma^2}{2}} e^{i\pi n}$$

$$- 2^n A^{(4)}_1 A^{(4)}_2 \, e^{-\frac{(X-X_*)^2 \sigma^2}{2}} e^{-i\pi n}$$

$$X_* = 4\pi \epsilon_1$$

Within support of the wavepacket, the amplitude takes the form

$$A_4^{(1)} A_4^{(2)} B(\alpha' k_D^2, \eta)$$

$\eta < 0$: Beta funtion does not admit a convergent expansion of propagators $1/(k_D^2 + n)$
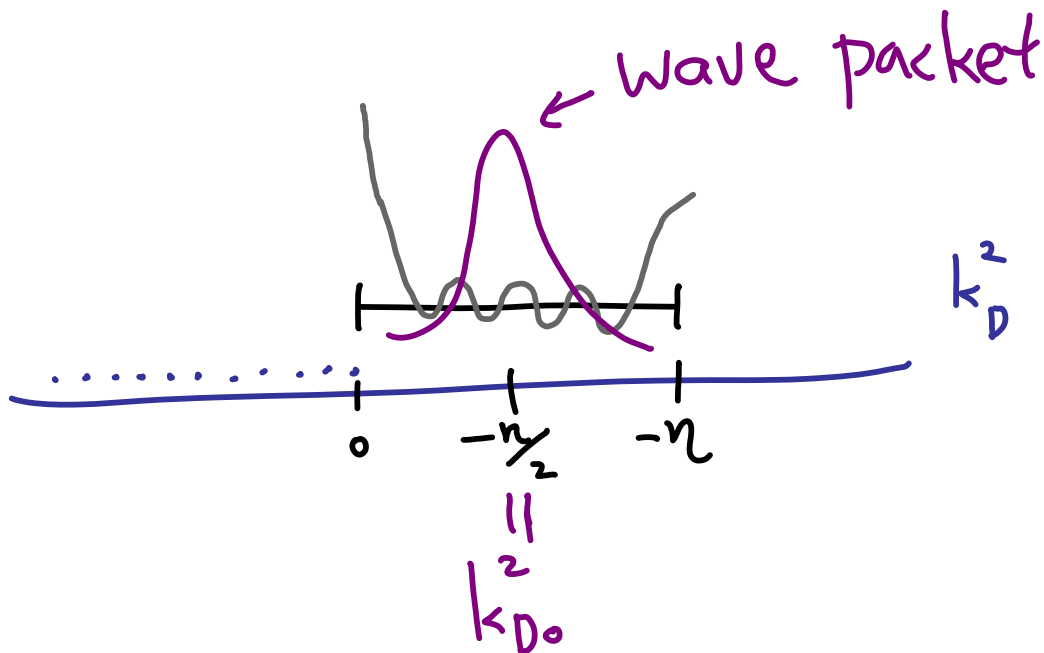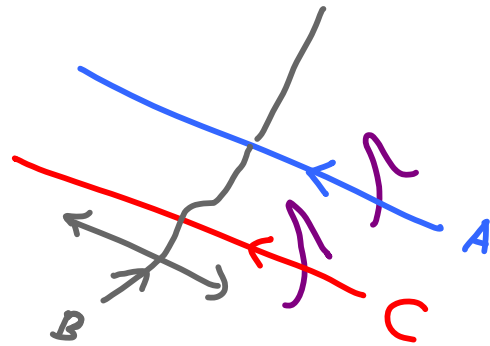
$$A_4^{(1)} A_4^{(2)} \times \frac{\sin \pi (\alpha' k_D^2 + \eta) \Gamma(\alpha' k_D^2) \Gamma(-\alpha' k_D^2 - \eta)}{\Gamma(-\eta) \sin \pi \eta}$$

$$e^{i\pi P_{\hat{B}}^+ \delta k_C^-} \quad \text{shifts support of } x^+$$

$$k_D^2 = \frac{K_{C\hat{C}}}{\alpha'} + 4 E_{\hat{B}}(E_C - E_{\hat{C}}) + \frac{(E_C q_{\hat{B}} + E_{\hat{B}} q_C)^2}{E_{\hat{B}} E_C} - \frac{(E_{\hat{C}} q_{\hat{B}} - E_{\hat{B}} q_{\hat{C}})^2}{E_{\hat{B}} E_{\hat{C}}}$$

$$= k_{D_0}^2 + 4 E \, \delta_{p_C}$$

$$q_A = q_c = 0$$

← wave packet

$$k_D^2$$

$$0 \qquad -\eta/2 \qquad -\eta$$

$$\underset{k_{D_0}^2}{\parallel}$$

$$\frac{A_{\text{QFT}}}{A_{\text{ST}}} \simeq \frac{1}{(-\eta/2)} \left(\frac{2}{e^{\pi^2 c_\sigma/2}} \right)^{-\eta} \ll 1$$

Could the ST answer be on tail of wavefunction? A priori, might take advantage of higher momentum-space amplitude elsewhere in p_c. But Gaussian wavepacket suppresses this, and dilaton tracer consistent with spreading interaction.

How does this compare to light cone prediction? Consistent:

$$\Delta X^+_{\text{spreading}} \sim \frac{k^+_D}{k^+_D k^-_D} \sim \frac{E_{\hat{B}}}{k^+_D k^-_D}.$$

$$\exp\left(-\text{const} \times \frac{\Delta X^+}{\Delta X^+_{\text{spreading}}}\right) \sim \exp\left(-\text{const} \times \frac{E_{\hat{B}}\alpha'}{E_{\hat{B}}/k^+_{D0}k^-_{D0}}\right) \sim 2^\eta$$

Also incorporating the dilaton g~exp(VX) into the light cone prediction reveals that it degrades for X~1/V. This behavior comes out of the S-matrix amplitude as well, along with expected power of g(Xspreading).

Closed string version: similar spreading regime, but overall s-dependence stronger:

$$\mathcal{A}_4 \sim \frac{s_{early,late}^{2-q^2\alpha'}}{q^2}$$ => growth

$$s_{early,late} = s_0 e^{\Delta t/2r_s}$$

$$Prob \sim \sum_{final\ states} \sim \frac{1}{N_{id}!} \int \prod_f \frac{dp_f d^{d-2}q_f}{2E_f} |\mathcal{A}(X^+)|^2$$
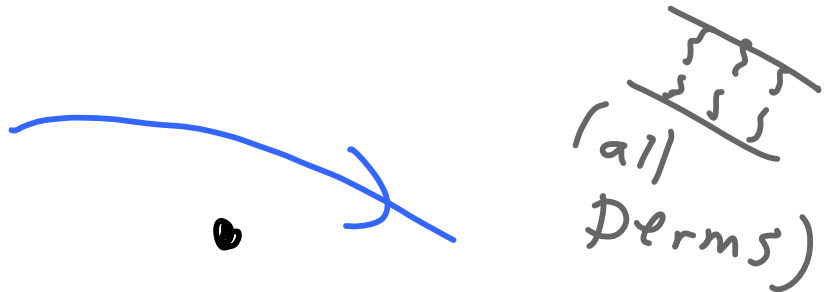
$$\smallsmile \quad s_{early-late}^2 G_N^2 \times f(q, E_0, r_s, \tilde{\eta}, \sigma_I, \dots)$$

=> need quantum corrections

Finite string coupling: light cone calculation invoked high-energy modes (shut off at finite g); Gross-Mende find scale of non-locality suppressed like 1/genus; if scatterers (bit of string + detector) hit their own r_Schwarzshild then breaks down.
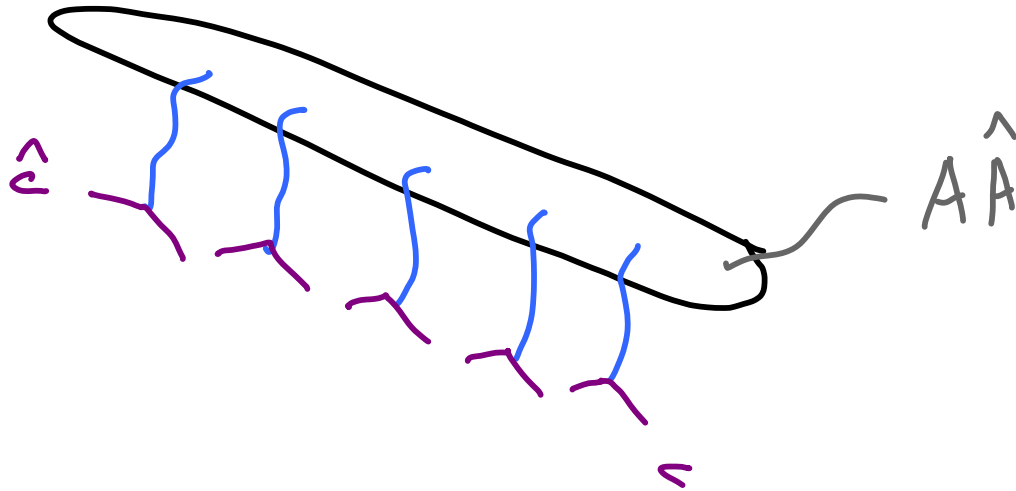
Next step: Eikonal scattering

GR version:

(all
Perms)

$$
\mathcal{A}_{Eikonal,GR} \sim G_N s^2 \int \frac{d^4 k_0}{k_0^2} \int d^4 x\, e^{-i(k_{C\hat{C}} - k_0)x} \sum_{N=0}^{\infty} \frac{(i\chi)^N}{(N+1)!}
$$

$$
= G_N s^2 \int \frac{d^4 k_0}{k_0^2} \int d^4 x\, e^{-i(k_{C\hat{C}} - k_0)x} \frac{e^{i\chi(x)} - 1}{\chi(x)}
$$

$$
\chi_{GR}(x) = G_N \int d^4 k\, e^{ik \cdot x} \frac{s^2}{k^2 - i\epsilon} \left[\frac{1}{-p_C \cdot k + i\epsilon} \frac{1}{p_B \cdot k + i\epsilon} + \frac{1}{-p_{\hat{C}} \cdot k - i\epsilon} \frac{1}{p_{\hat{B}} \cdot k - i\epsilon} \right.
$$
$$
\left. - \frac{1}{-p_C \cdot k + i\epsilon} \frac{1}{p_{\hat{B}} \cdot k - i\epsilon} - \frac{1}{-p_{\hat{C}} \cdot k - i\epsilon} \frac{1}{p_B \cdot k + i\epsilon} \right]
$$

$N_* \sim G_N s$

$k_{\perp *} \sim \frac{1}{x_{\perp}}$

Building block (includes permutations):

$$\prod_r \frac{K_{C\hat{B}}^{2-K_{C\hat{C},r}/2}}{K_{C\hat{C},r}} \int d^2 z_{\hat{A}} |z_{\hat{A}}|^{K_{\hat{B}\hat{A}}/2} |1-z_{\hat{A}}|^{K_{A\hat{A}}/2} \int \prod_i d^2 z_{(C\hat{C})_i} \prod_{l\neq m} |z_{(C\hat{C})_l} - z_{(C\hat{C})_m}|^{K_{(C\hat{C})_l(C\hat{C})_m}/2}$$

$$\prod_j |z_{(C\hat{C})_j}|^{K_{\hat{B}(C\hat{C})_j}/2 - 2 + K_{C\hat{C},j}/4} |1 - z_{(C\hat{C})_j}|^{K_{(C\hat{C})_j A}/2} |z_{\hat{A}} - z_{(C\hat{C})_j}|^{K_{(C\hat{C})_j \hat{A}}/2}$$

$$K_{\hat{B}\hat{A}}^{-K_{A\hat{A}}/2} \prod_i \frac{K_{C\hat{B}}^{2-K_{C\hat{C},i}/2}}{K_{C\hat{C},i}} \prod_{j=1}^{n} F(\alpha' k_{D_j}^2/2, \tilde{\eta}_j)$$

Eikonal obtained by joining C's, integrating over momenta.

$$A(X_o^+)\Big|_{\text{N-loop}} = \int \prod_j^N dk_j^- \, e^{-(\sum k^- - p_{C0}^-)^2/2\sigma_0^2} e^{iX_0^+ \sum k^-} \prod_j^N F(k_j)$$

This contains a symmetric contribution which is a product of the tree-level results, each with $-\eta_j \sim 1/N$.

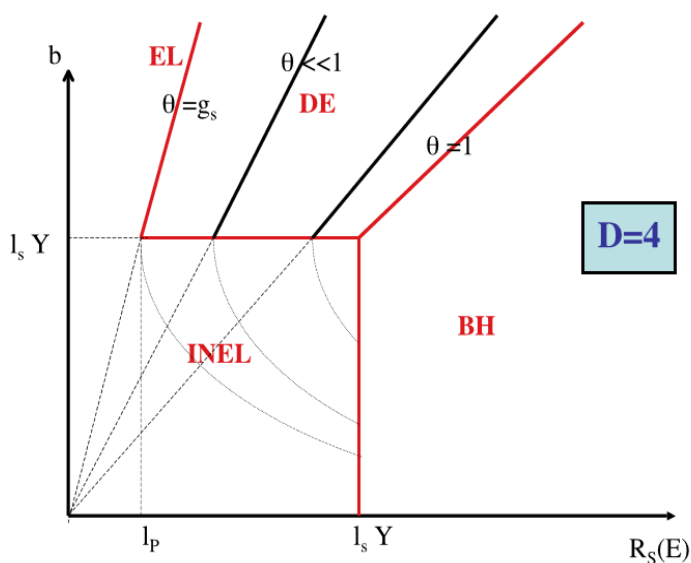$$N_* \sim G_N S, \quad -\eta_j \gg 1$$

$$\Rightarrow G_N S < \sim \eta$$

$$\|$$

$$G_N S_0 \, e^{\frac{\Delta t}{2r_s}} \qquad S_0 \sim \frac{1}{r_s^2}$$

$$\Rightarrow \Delta t < 2r_s \log\left(\frac{-\eta}{G_N S_0}\right)$$

$$\sim \text{scrambling time} \sim r_s \log S$$

Veneziano et al have been developing a phase diagram for high energy scattering. A longer-term goal is to generalize the whole diagram to incorporate longitudinal effect (probed at higher points as described here, or in a BH background with the large non-local s).

With that in hand, we could obtain an accurate estimate for string theory interactions near black holes. (So far, other approaches to BH info more dramatic (e.g. traversable wormholes carrying out the info?))

Summary:

Longitudinally separated interactions pass strong S-matrix test, at scale ~ $E\alpha'$.

This raises interesting questions and suggests several applications, including interactions between separated horizon infallers.

Closed string version grows with time, loop corrections controllable until ~scrambling time (full effect still TBD).