Horizon hair of extremal black holes and measurements at null infinity

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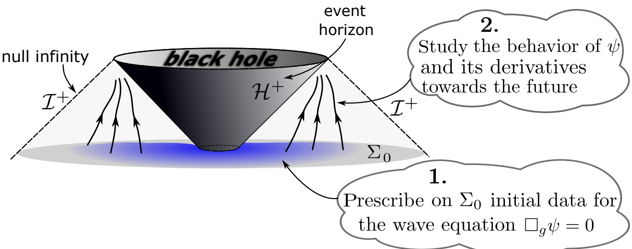
Evolution of scalar fields

- Scalar fields: Investigate the evolution of solutions to wave equation

$$\Box_g \psi = 0$$

  on black hole backgrounds (Schwarzschild, Kerr, Reissner–Nordström)



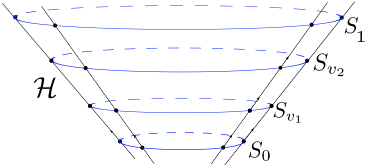
- Motivation: In harmonic gauge $\Box_g x^\mu = 0$ the vacuum equations take the form

$$\Box_g g_{\mu\nu} = N_{\mu\nu}(g, \partial g).$$

  Hence the wave equation serves as a (necessary) toy model in studying the dynamics of Einstein equations.
- Goal:
  - Upper bounds for stability considerations
  - Lower bounds for strong cosmic censorship
- This talk: Emphasis on conservation laws, asymptotics and physical consequences

Conservation laws along characteristic hypersurfaces

Let S_v be a foliation with section of a null hypersurface $\mathcal{H}$.



Then roughly speaking a conservation law consists of integrals of the form

$$\int_{S_v} F(\psi, D^a\psi)$$

which are independent of v for all scalar fields ψ satisfying the wave equation.

Late-time tails on sub-extremal black holes

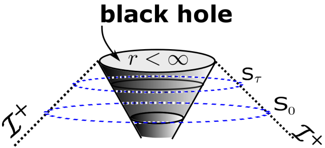
Previous mathematical works

Very active research area in the past decade.

- Main difficulties: Low frequencies, superradiant, trapping, redshift
- Contributors: Dafermos, Rodnianski, Andersson, Tataru, Moschidis, Blue, Holzegel, Shlapentokh-Rothman, Dyatlov, Häfner, Bony, Smulevici, Klainerman, Ionescu, Tohaneanu, Sterbenz, Soffer, Schlue, Luk, Finster, Kamran, Smoller, Yau, Donninger, Schlag, Vasy, Hintz, Metcalfe, Wald, ...
- Lower bounds were first proved in the work of Luk–Oh.
- All methods break down at the extremal case.

The Newman–Penrose constant

- The Newman–Penrose constant gives rise to a conservation law along null infinity.



The constant is equal to

$$NP[\psi] = \int_{S_\tau} \lim_{r\to\infty} r^2 \cdot \partial_v(r\psi)$$

- However, $NP[\psi] = 0$ for compactly supported data. Then, generically,

$$NP[\partial_t^{-1}\psi] \neq 0$$

where $\partial_t^{-1}\psi$ is canonically defined as long as $\partial_t \neq 0$.
- The constant $NP[\partial_t^{-1}\psi]$ can be explicitly computed using the initial data of ψ.
- Denote $I^{(1)}[\psi] := NP[\partial_t^{-1}\psi]$.
- $I^{(1)}$ is the unique obstruction to inverting T^2.

Late-time asymptotics

Theorem (Angelopoulos, A., Gajic)

If ψ is a solution to the wave equation on a sub-extremal Reissner–Nordström space-time with smooth compactly supported initial data then

Asymptotics in the exterior region		
$\psi\|_{\mathcal{H}}$	$\psi\|_{r=R}$	$r\psi\|_{\mathcal{I}}$
$8I^{(1)}[\psi]\cdot\tau^{-3}$	$8I^{(1)}[\psi]\cdot\tau^{-3}$	$-2I^{(1)}[\psi]\cdot\tau^{-2}-8MI^{(1)}[\psi]\log\tau\cdot\tau^{-3}$

Comments:

- $$I^{(1)}[\psi]=\frac{M}{4\pi}\int_{\{t=0\}\cap S_{\mathrm{BF}}}\psi\,d\Omega+\frac{M}{4\pi}\int_{\{t=0\}}\frac{1}{1-\frac{2M}{r}}\partial_t\psi\,r^2drd\Omega.$$
- Sharp lower and upper pointwise bounds.
- $I^{(1)}[\psi]$ related to the quantity $\mathcal{L}$ of Luk–Oh.
- Correlated asymptotics along $\mathcal{H}^+$ ($\psi\sim 8I^{(1)}[\psi]\cdot\tau^{-3}$) and $\mathcal{I}^+$ ($r\psi\sim-2I^{(1)}[\psi]\cdot\tau^{-2}$).
- Leading order asymptotics recover work of Leaver.
- Precise logarithmic corrections along $\mathcal{I}^+$ appear to be new.
- We further obtain $(2\ell+3)$-asymptotics.

Late-time tails (or late-time tales) for extremal black holes

Why extremal black holes?

- Mass minimizers
- Applications in supersymmetry, quantum gravity, string theory
- Electromagnetic and gravitational signatures
- Turbulent gravitational behavior
- Vast astronomical evidence for near-extremal black holes.
- Rees et al. (*The distribution and cosmic evolution of massive black hole spins*, Astrophys. J.) report that "the spin distribution is heavily skewed toward fast-rotating Kerr black holes" and that "about 70% of all stellar black holes at all epochs are maximally rotating". Gas accretion dominant effect and spins black holes up.

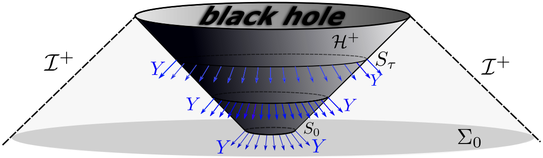
Firstly, we have the following

Proposition (A.)

If ψ satisfies the wave equation on extremal Reissner–Nordström then the integral

$$H[\psi] = -\int_{S_\tau} \left(Y\psi + \frac{1}{2M}\psi \right) dvol$$

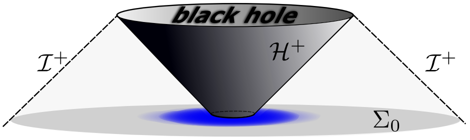
is independent of τ. Here Y is transversal to the horizon.



- For smooth solutions ψ we have $H[T\psi] = 0$. Hence H is an obstruction to inverting T.

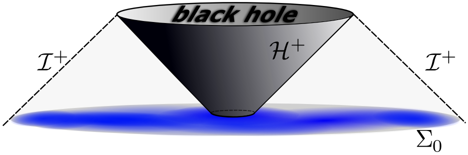
“Outgoing radiation”

Solutions ψ with $H[\psi] \neq 0$ and $NP[\psi] = 0$



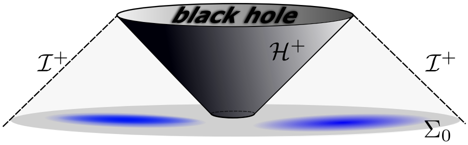
"Initially static moment"

Solutions ψ with $H[\psi] \neq 0$ and $NP[\psi] \neq 0$



“Ingoing radiation”

Solutions ψ with $H[\psi] = 0$ and $NP[\psi] = 0$



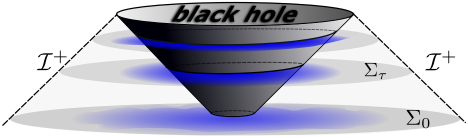
$H[\psi]$ as a "horizon hair"

- Outgoing perturbations and perturbations with an initially static moment ($H[\psi] \neq 0$) satisfy along the event horizon:

  1) **Non-decay**: $Y\psi \to -\frac{1}{M}H[\psi]$
  2) **Blow-up**: $YY\psi \to \frac{1}{M^3}H[\psi] \cdot \tau$

- $H[\psi]$: "horizon" "hair" since

  1) Energy density measured by incoming observers: $\boldsymbol{T}_{rr}[\psi] \sim H[\psi]$ where $\boldsymbol{T}$ is the E-M tensor,
  2) $|Y^k\psi|, |\boldsymbol{T}_{rr}[\psi]| \leq 0$ away from the horizon.



- Generic ingoing perturbations: $|YYY\psi| \to \infty$, as $\tau \to +\infty$.

- Later extensions/applications by: Reall, Murata, Casals, Zimmerman, Gralla, Tanahashi, Bizon, Lucietti, Angelopoulos, Gajic, Ori, Sela, Tsukamoto, Kimura, Harada, Hadar, Dain, Dotti, Godazgar, Burko, Khanna, Bhattacharjee, Chow, Berti et al, Cardoso et al,...

Late-time asymptotics

Theorem (Angelopoulos, A.,Gajic)

The following asymptotics hold on ERN:

	Asymptotics along the event horizon	
Perturbation	*outgoing data*	*ingoing data*
$\psi\|_{\mathcal{H}}$	$\boldsymbol{2H\cdot\tau^{-1}}$	$\boldsymbol{-2H^{(1)}\cdot\tau^{-2}}$
$Y\psi\|_{\mathcal{H}}$	$-\frac{1}{M}\cdot H$	$\boldsymbol{\frac{2}{M^2}\cdot H^{(1)}\cdot\tau^{-2}}$
$YY\psi\|_{\mathcal{H}}$	$\frac{1}{M^3}\cdot H\cdot\tau$	$\frac{1}{M^3}\cdot H^{(1)}$
$YYY\psi\|_{\mathcal{H}}$	$-\frac{3}{2M^5}\cdot H\cdot\tau^2$	$-\frac{3}{M^5}\cdot H^{(1)}\cdot\tau$

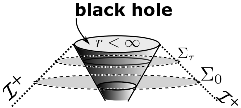
- H registers in the asymptotics for ψ.
- Here $H^{(1)}=H[T^{-1}\psi]$. It is well-defined for ingoing perturbations.
- Asymptotics on $\mathcal{H}^+$ confirm numerical results of Murata–Reall–Tanahashi and is consistent with decay rates of Blaskley–Burko, Ori–Sela and Casals–Gralla–Zimmerman.
- Asymptotics (with log corrections) on the event horizon are important for dynamics in the *interior* of extremal black holes–C^2 extendibility. (Gajic, Reall et al., Gajic–Luk).

Late-time asymptotics

Theorem (Angelopoulos, A.,Gajic)

The following asymptotics hold on ERN:

	Asymptotics away from the event horizon	
Data	$\psi\|_{r=R}$	$r\psi\|_{\mathcal{I}}$
outgoing	$\frac{4M}{r-M}H\cdot\tau^{-2}$	$\boldsymbol{\left(4MH-2I^{(1)}\right)\cdot\tau^{-2}}$
static moment	$4\left(NP+\frac{M}{r-M}H\right)\cdot\tau^{-2}$	$2\cdot NP[\psi]\cdot\tau^{-1}$
ingoing	$-8\left(I^{(1)}+\frac{M}{r-M}H^{(1)}\right)\cdot\tau^{-3}$	$-2I^{(1)}\cdot\tau^{-2}$



- Here $I^{(1)}=NP[T^{-1}\psi]$ and $H^{(1)}[\psi]=H[T^{-1}\psi]$.
- $H[\psi]$ registers in the asymptotics away from $\mathcal{H}$, even on null infinity $\mathcal{I}$.
- $\frac{4M}{r-M}$ is the static solution.
- For outgoing perturbation ψ, $T^{-1}\psi$ is singular on $\mathcal{H}^+$: its local energy is infinite.
- Asymptotics for $r\psi|_{\mathcal{I}}$ were not known in physics literature.

Measuring the horizon hair H from null infinity

- In principle, precise asymptotics allow to observe/measure the horizon instability from afar.

Let's consider outgoing radiation.

- Along $r = R > M$: We have a slower decay rate if $H \neq 0$. In fact,

$$\boxed{H[\psi] = \frac{R-M}{4M} \cdot \lim_{\tau \to \infty} \tau^2 \cdot \psi|_{r=R}}$$

- Along $\mathcal{I}^+$: We have the same decay rate for the radiation field, but the horizon hair registers in the asymptotics $r\psi|_{\mathcal{I}} \sim \left(4MH - 2I^{(1)}\right) \cdot \tau^{-2}$. In fact, it turns out that $I^{(1)} = \frac{M}{4\pi} \int_{\mathcal{I}^+ \cap \{\tau \geq 0\}} r\psi \, d\Omega d\tau$ which yields

$$\boxed{H[\psi] = \frac{1}{4M} \lim_{\tau \to \infty} \left(\tau^2 \cdot (r\psi)|_{\mathcal{I}}\right) + \frac{1}{8\pi} \int_{\mathcal{I}^+ \cap \{\tau \geq 0\}} r\psi|_{\mathcal{I}} \, d\Omega d\tau}$$

- We conclude that for extremal black holes information "leaks" from the event horizon to null infinity.

Comparison with sub-extremal tails

For ERN:

$$H[\psi] = \frac{1}{4M} \lim_{\tau \to \infty} \left(\tau^2 \cdot (r\psi)|_{\mathcal{I}} \right) + \frac{1}{8\pi} \int_{\mathcal{I}^+ \cap \{\tau \geq 0\}} r\psi|_{\mathcal{I}} \, d\Omega d\tau$$

For sub-extremal RN we have:

- The RHS vanishes for sub-extremal RN!
- Specifically, we have

$$\lim_{\tau \to \infty} \left(\tau^2 \cdot (r\psi)|_{\mathcal{I}} \right) = -\frac{M}{2\pi} \int_{\mathcal{I}^+ \cap \{\tau \geq 0\}} r\psi|_{\mathcal{I}} \, d\Omega d\tau,$$

$$\lim_{\tau \to \infty} \left(\tau^3 \cdot \psi|_{r=R} \right) = \frac{2M}{\pi} \int_{\mathcal{I}^+ \cap \{\tau \geq 0\}} r\psi|_{\mathcal{I}} \, d\Omega d\tau,$$

$$\lim_{\tau \to \infty} \left(\tau^3 \cdot \psi|_{\mathcal{H}} \right) = \frac{2M}{\pi} \int_{\mathcal{I}^+ \cap \{\tau \geq 0\}} r\psi|_{\mathcal{I}} \, d\Omega d\tau,$$

- Late time tails are dictated by the weak-field dynamics, namely by dynamics at very large r.
- Integral of the radiation field had been used by Luk–Oh for lower bounds on sub-extremal RN.

Physics Literature

- Work by Reall, Murata and Tanahashi suggests that perturbations of initial data of extremal R–N in the context of the Cauchy problem for the Einstein–Maxwell-scalar field equations exhibit a version of the horizon instability.
- Work by Casals–Gralla–Zimmerman and subsequently by Hadar–Reall obtained that the decay rate for non-zero azimuthal frequencies along the event horizon on extremal Kerr is $\frac{1}{\sqrt{\tau}}$ and for the first-order transversal derivative is $\sqrt{\tau}$ (amplified instability).

Thank you!