

Black hole instabilities and violation of the weak cosmic censorship in higher dimensions

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[Phys.Rev.Lett. 116 \(2016\) no.7, 071102](#)
[Phys.Rev.Lett. 118 \(2017\) no.15, 151103](#)
work in progress



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Why gravity in higher D ?

- Study fundamental aspects of gravity in new settings
- String theory, AdS/CFT
- GR simplifies in the large D limit
- New gravitational physics in $D > 4$:
 1. Gravitational instabilities [[Gregory and Laflamme](#)]
 2. New black hole topologies [[Emparan and Reall](#); [Schoen and Galloway](#)]

Outline of the talk

- Motivation: the weak cosmic censorship conjecture
- Black ring instabilities
- Rotating spherical black hole instabilities
- Summary and conclusions

The weak cosmic censorship conjecture

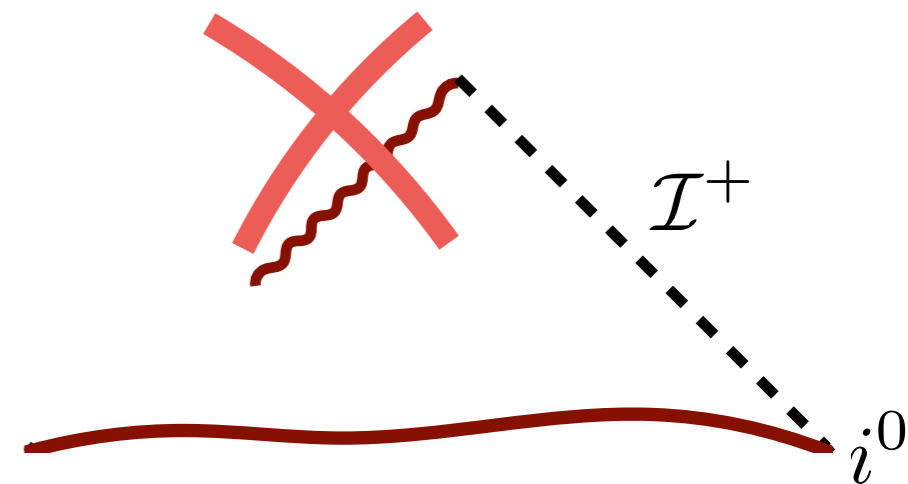
- GR has a well-posed initial value problem [\[Choquet-Bruhat; Choquet-Bruhat and Geroch; Sbierski\]](#)
- Singularity theorems in GR: singularities form generically [\[Penrose; Hawking and Penrose\]](#)
- If singularities form generically, does GR have any predictive power at all?
- What kind of singularities form generically in dynamical evolution?

The weak cosmic censorship conjecture

*“**Generic** asymptotically flat initial data have a maximal future development possessing a complete future null infinity”*

[Penrose; Geroch and Horowitz; Christodoulou]

- If a black hole is unstable, can the singularity inside become visible during the evolution?



The Gregory-Laflamme instability for black strings

- Black strings: black hole solution of the Einstein vacuum equations in $M_4 \times S^1$

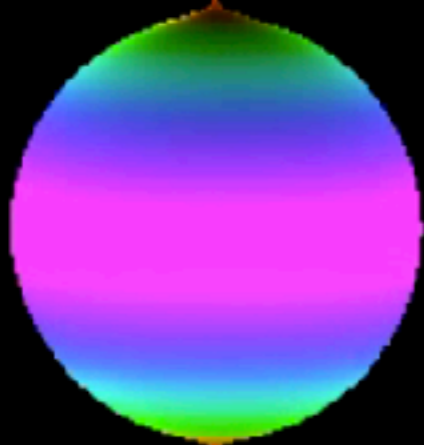
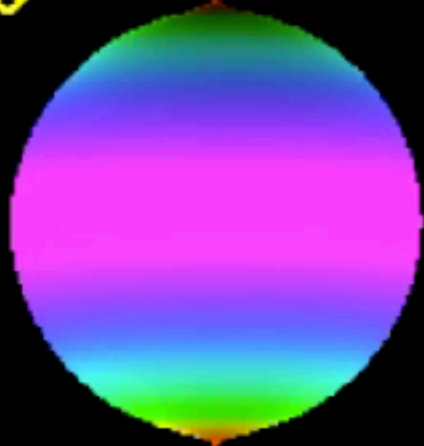
$$ds^2 = - \left(1 - \frac{2M}{r} \right) dt^2 + \frac{dr^2}{1 - \frac{2M}{r}} + r^2 d\Omega_{(2)}^2 + dz^2 \quad z \sim z + L$$



- If $M/L \lesssim O(1)$ black strings are unstable to develop ripples along the compact extra dimension [\[Gregory and Laflamme\]](#)



$t=228.290$



4.09



0.0035

- The horizon develops a fractal structure
- Self-similar process
- The black string breaks in finite asymptotic time
- No fine-tuning is required
- The weak cosmic censorship conjecture may be false in spaces with compact extra dimensions

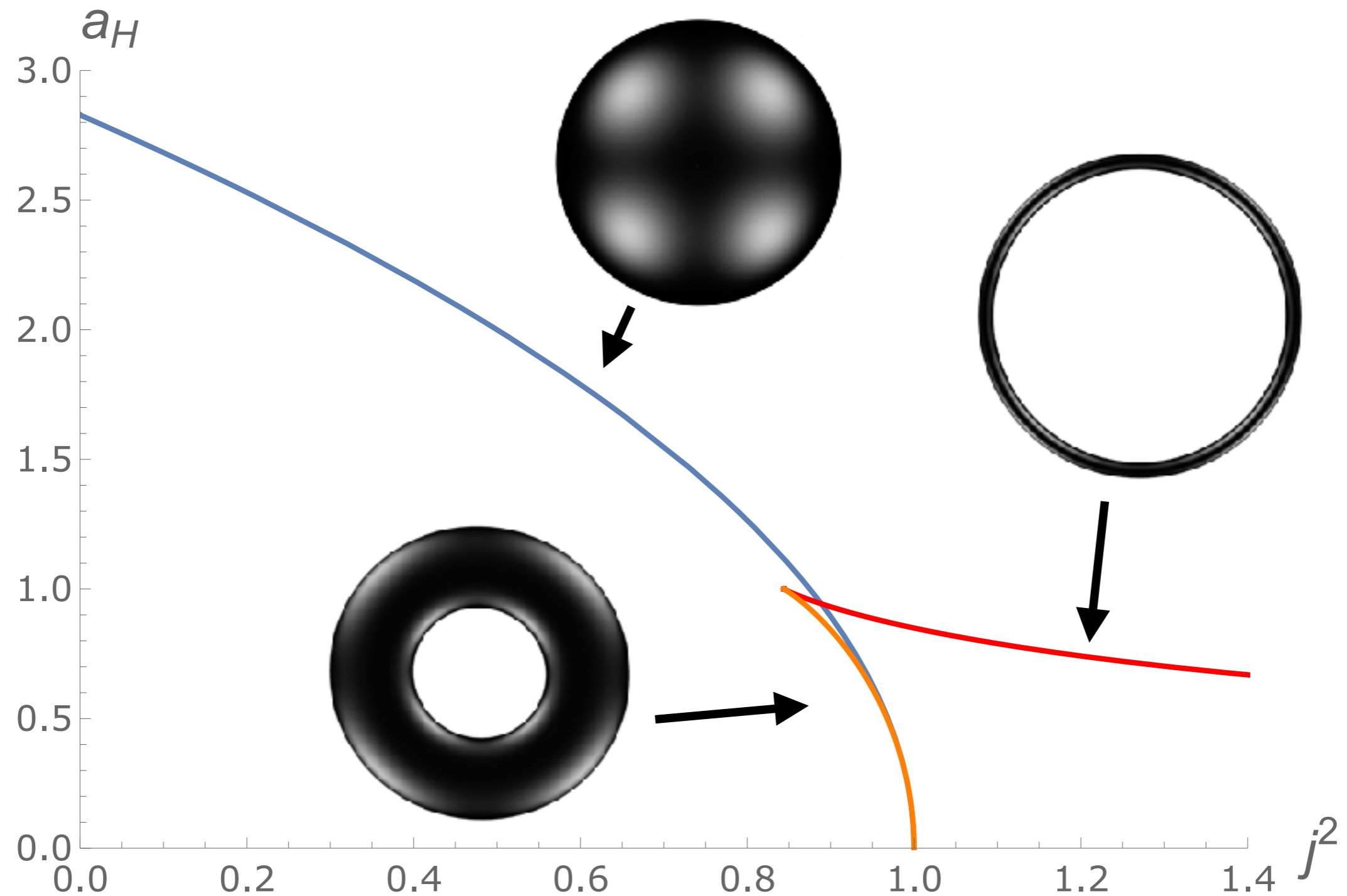
[Lehner and Pretorius]

Can the weak cosmic censorship conjecture be violated around higher dimensional asymptotically flat black hole spacetimes?

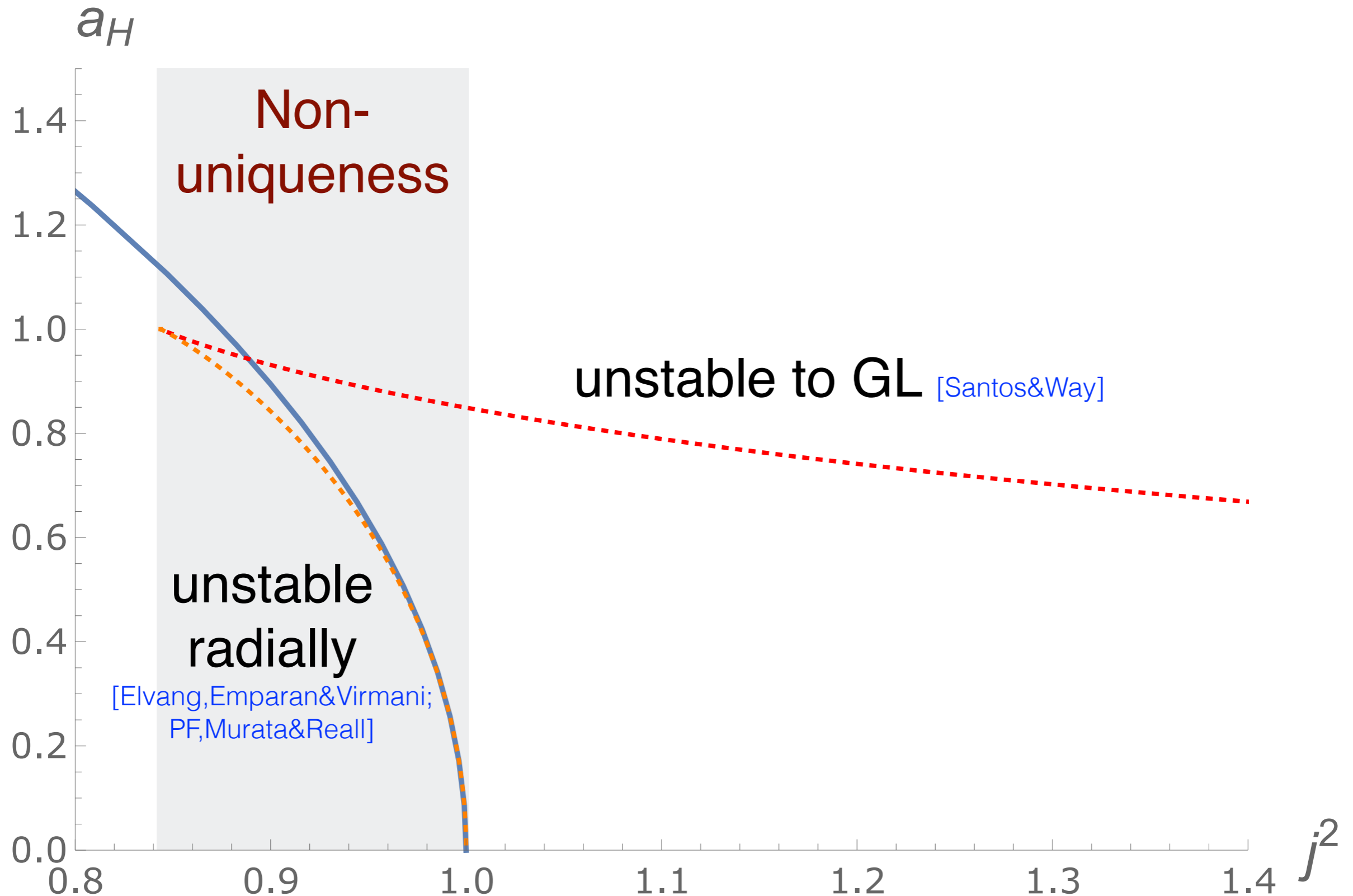
Black ring instabilities

Black hole phases in 5D

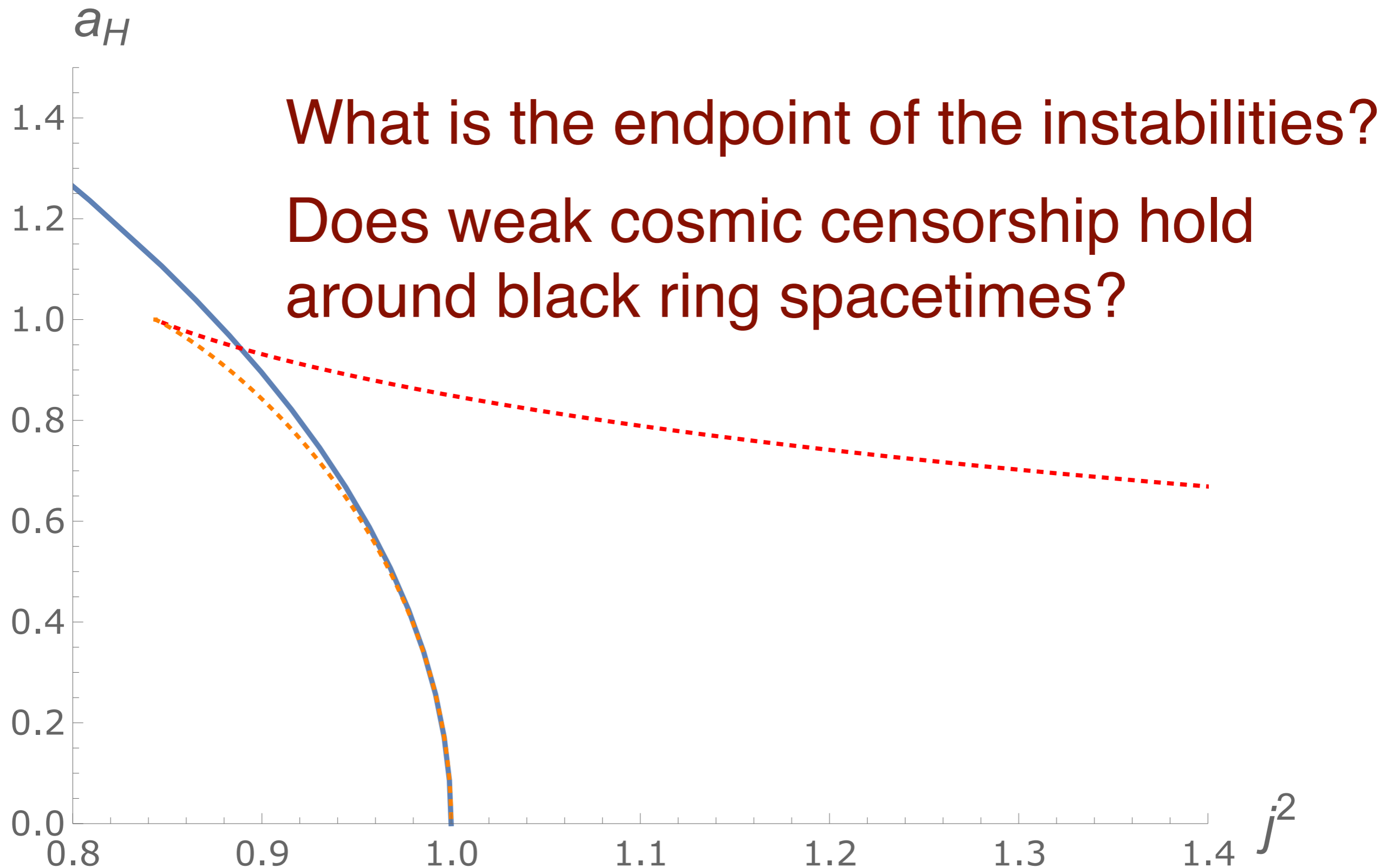
$M = 1$



Black hole phases in 5D

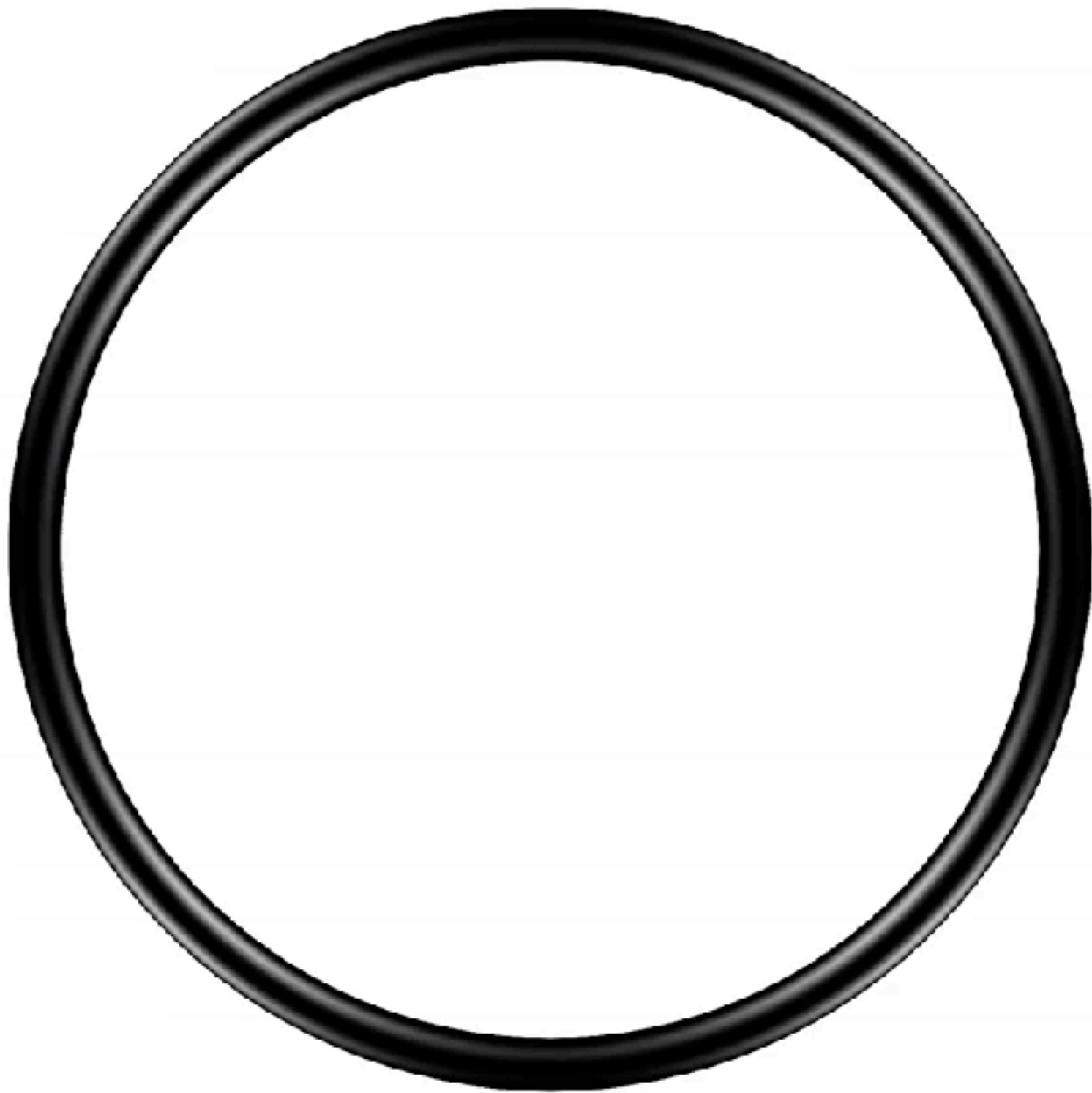


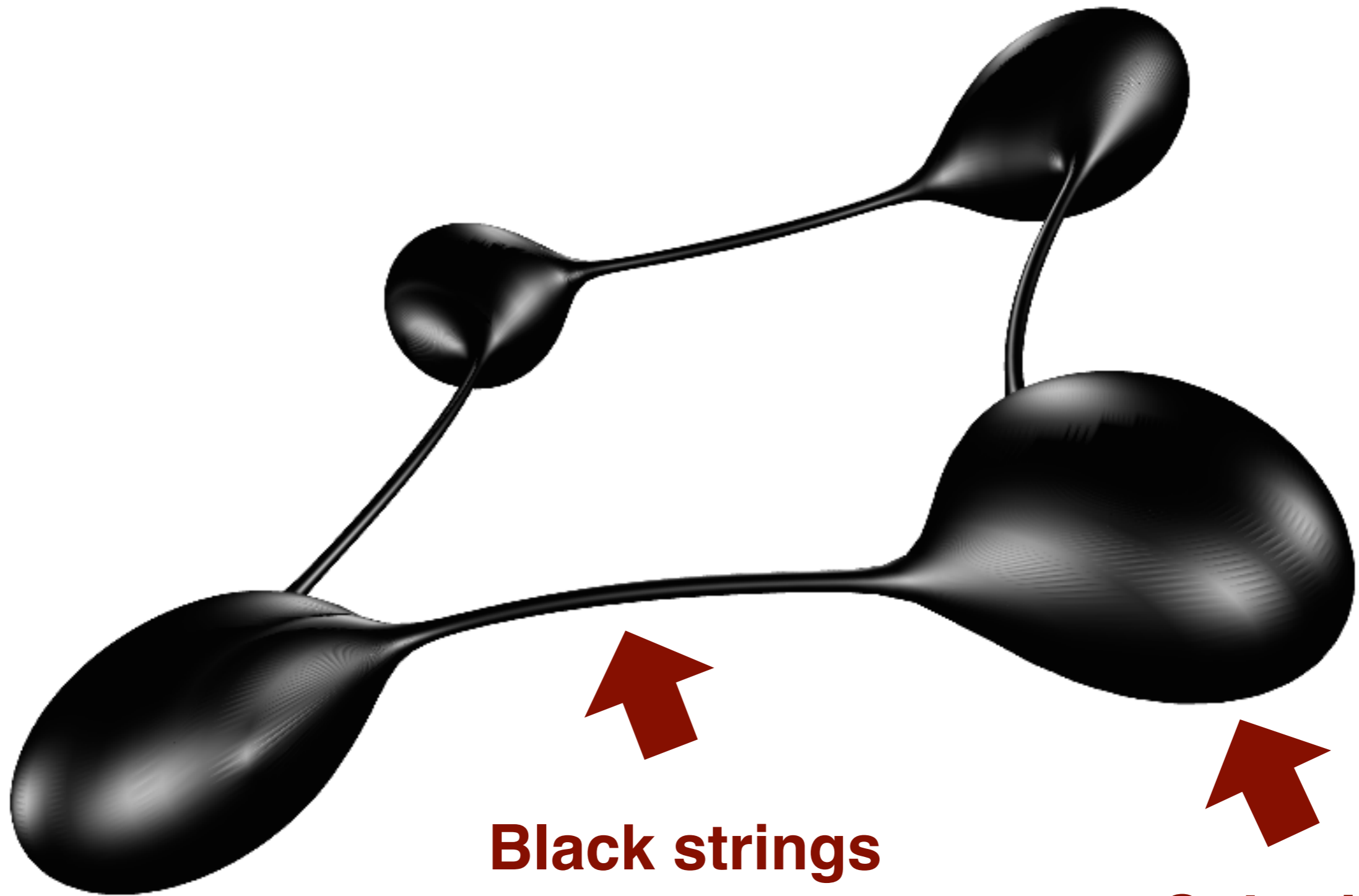
Black hole phases in 5D











Black strings

**Spherical
black holes**

- However the computations were very expensive (it's a 3+1 problem) and the understanding of the endpoint was limited:
 - Time-scale of the pinch-off could not be estimated
 - Is the process self-similar as in black strings?

Can we understand the details of the Gregory-Laflamme instability in asymptotically flat spaces?

Rotating spherical black hole instabilities

Myers-Perry BHs in $D \geq 6$

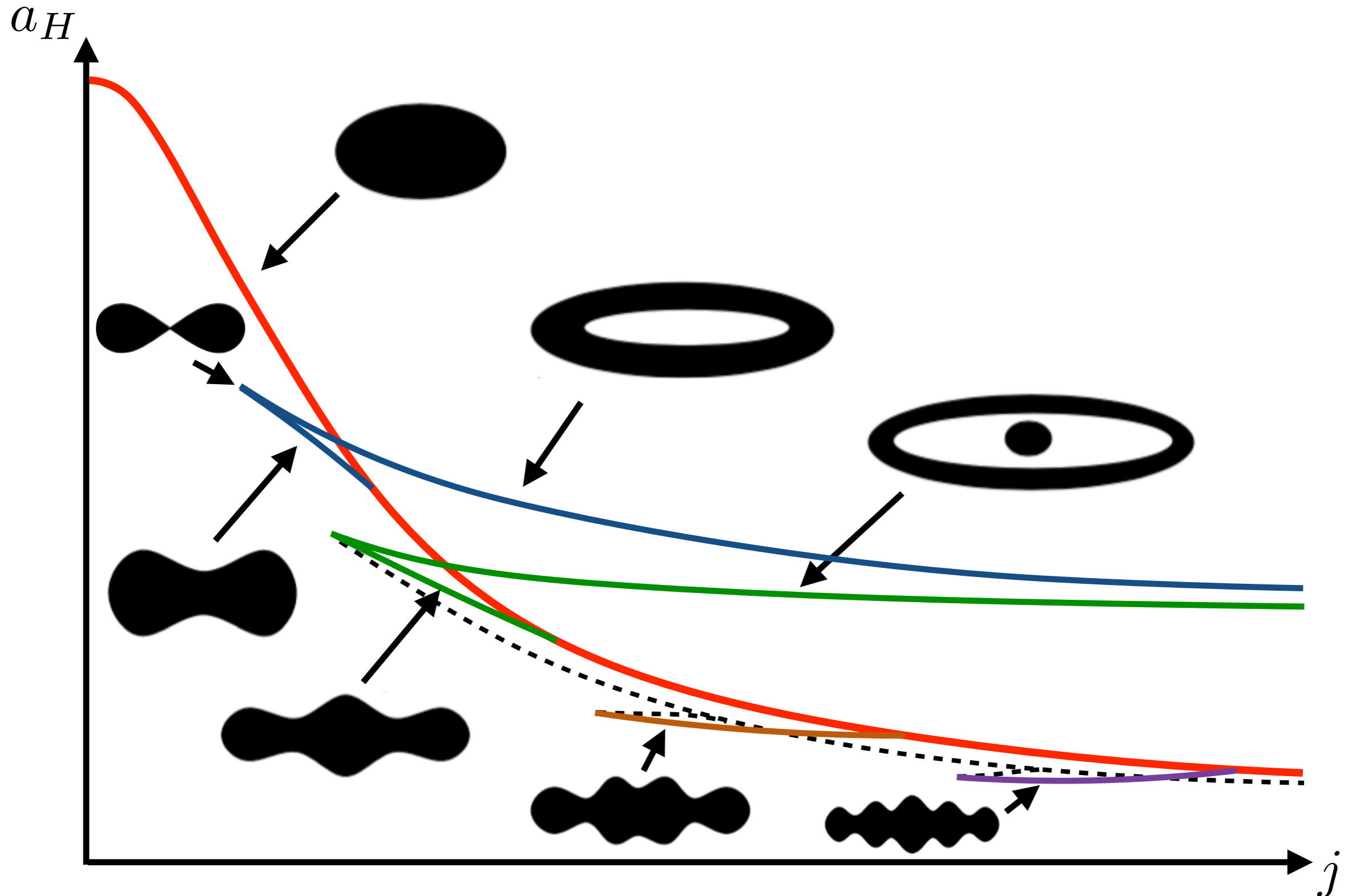
- The higher dimensional analogues of the Kerr BH:

$$ds^2 = - dt^2 + \frac{\mu}{r \Sigma} (dt - a \sin^2 \theta d\phi)^2 + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 \\ + (r^2 + a^2) \sin^2 \theta d\phi^2 + r^2 \cos^2 \theta d\Omega_{(D-4)}^2$$

$$\Sigma = r^2 + a^2 \cos^2 \theta \quad \Delta = r^2 + a^2 - \frac{\mu}{r^{D-5}} \quad \text{[Myers and Perry]}$$

- In $D \geq 6$ MP black holes can rotate arbitrarily fast
- In the limit $a \rightarrow \infty$, MP black holes resemble black membranes, which are unstable under the Gregory-Laflamme instability [Emparan and Myers]

Black hole phases in $D \geq 6$



[Emparan and Myers; Emparan et al., PF et al., Dias et al.,...]



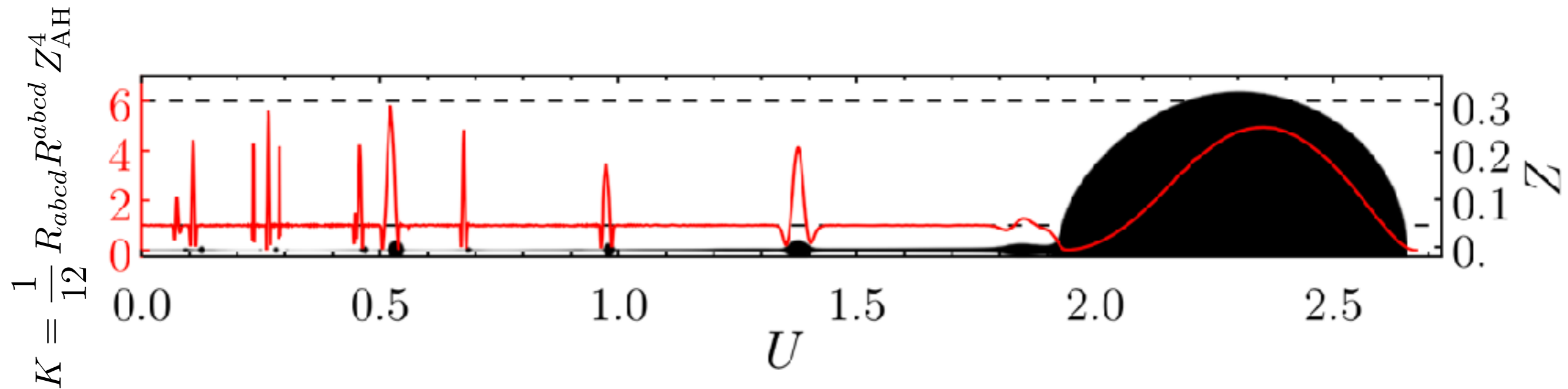
$$t/\mu^{\frac{1}{3}} = 30.0000$$





10,000 thinner than the original black hole!!!

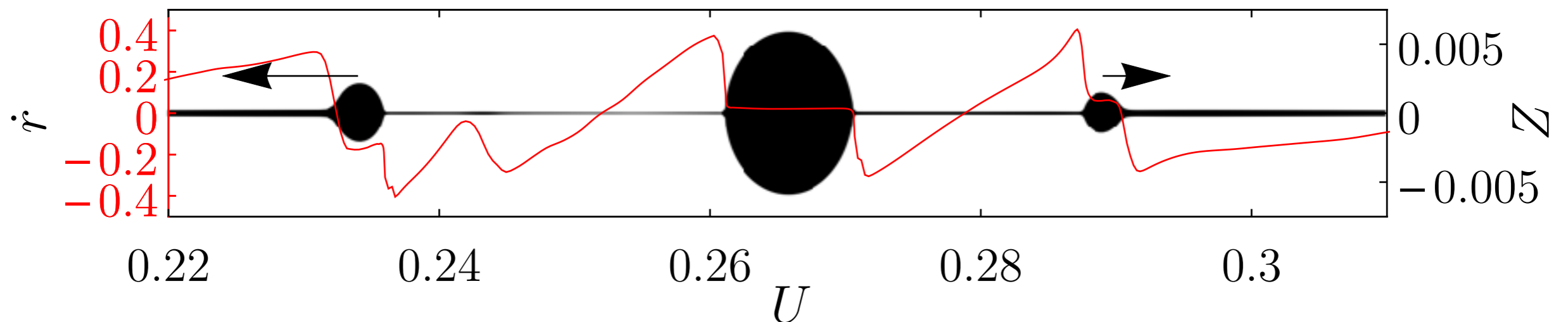
Evolution



- The local geometry is well approximated by a sequence of black rings connected by black membranes
- The outermost ring carries most of the mass and angular momentum

Evolution

- Differences between the dynamics of black strings and ultra spinning MP black holes:
 - Boundary effects are important initially
 - Centrifugal force: non-uniform membrane sections
 - Motion of higher generation rings

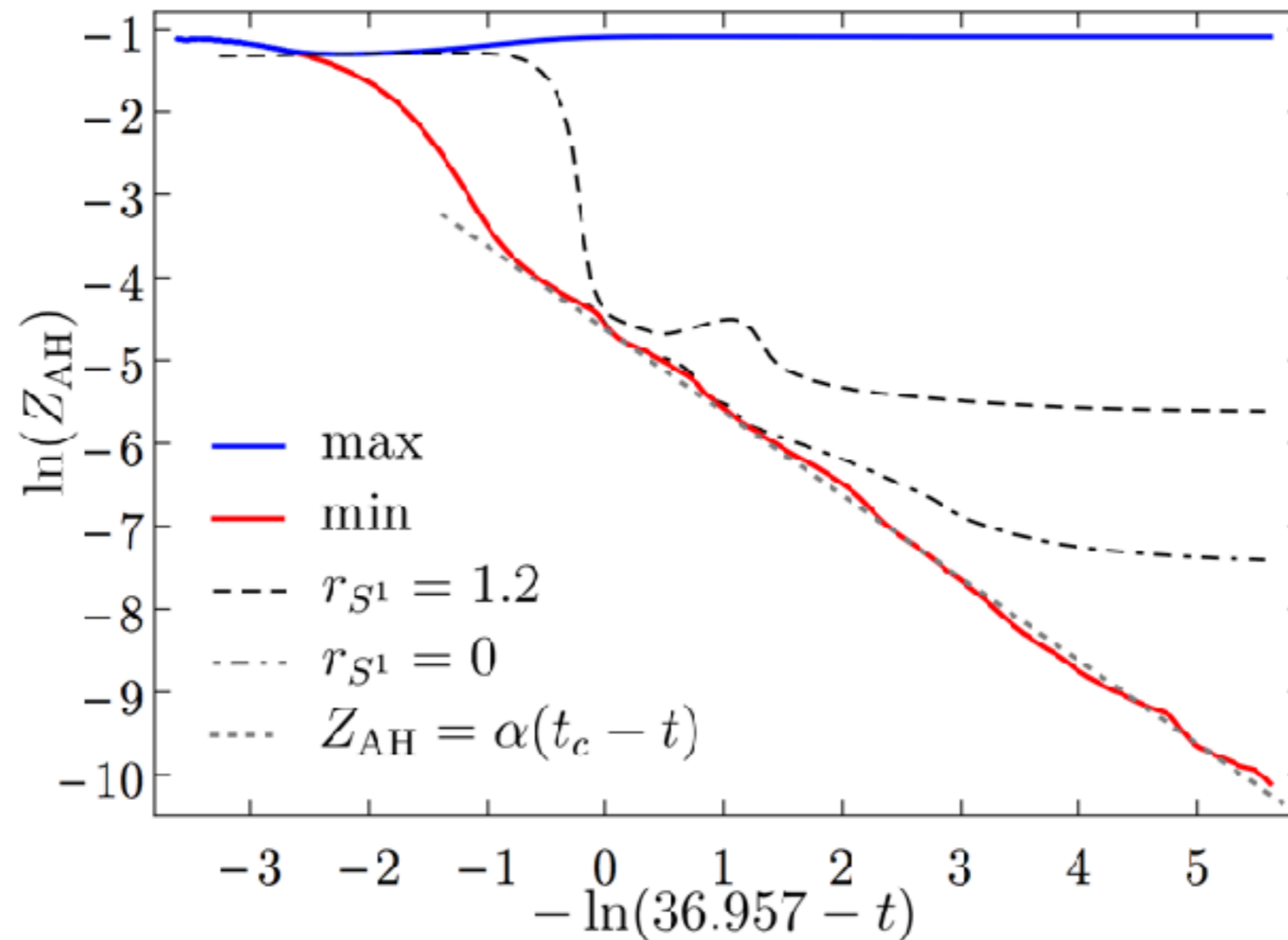


Evolution

The evolution of the ultra spinning instability of MP black holes is NOT self-similar

Evolution

- The minimum thickness follows a scaling law: $Z_{\text{AH}} = \alpha(t_c - t)$

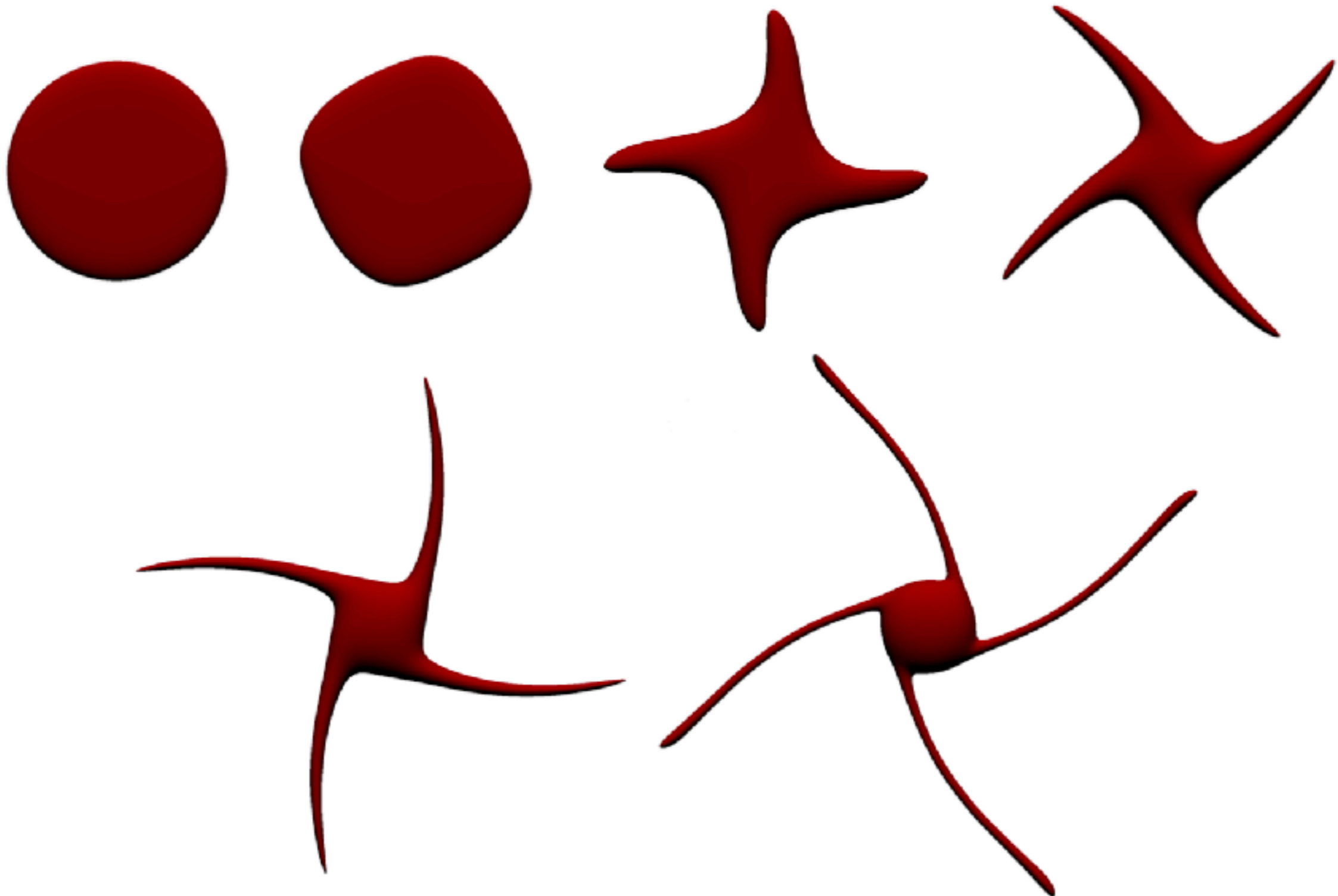


Summary and Conclusions

Summary and Conclusions

- Black rings and ultraspinning MP black holes are unstable and the instability evolves into a naked singularity in finite asymptotic time
- The weak cosmic censorship conjecture around ultraspinning MP black holes and black rings may be false
- This is generic in higher dimensions

- Evolution of non-axisymmetric instabilities of spherical black holes



- **Conjecture 1**

The Gregory-Laflamme instability is the only mechanism that GR has to change the horizon topology

- **Conjecture 2**

The only stable black hole in $D > 4$ is the Myers-Perry solution with $J/M^{D-3} \lesssim O(1)$



Thank you for your attention!